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## Why don't people pay attention? Endogenous sticky information in a DSGE Model

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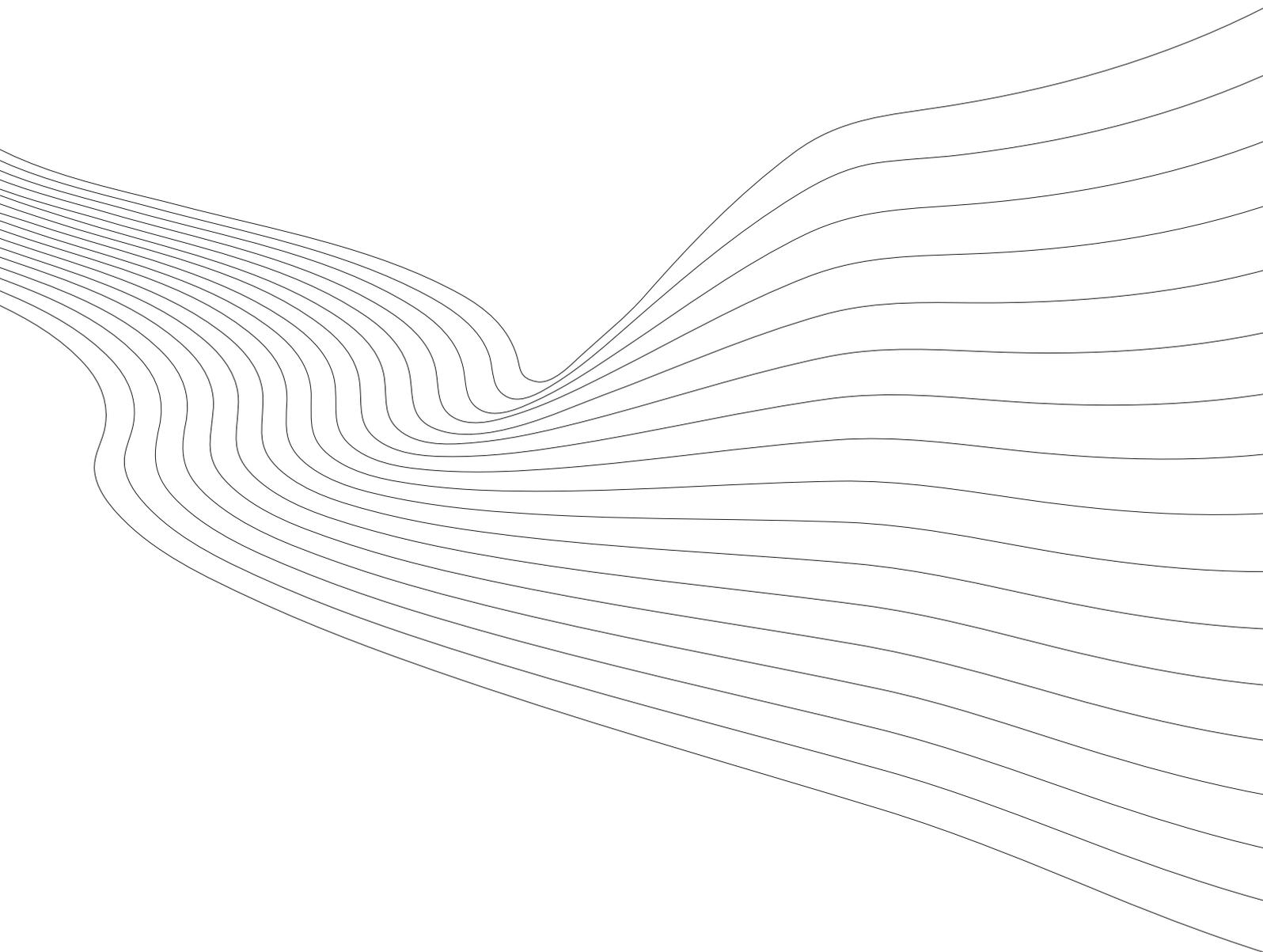
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# KOF Working Papers

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# Why don't people pay attention? Endogenous Sticky Information in a DSGE Model

Lena Dräger\*

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## Abstract

Building on the models of sticky information, we endogenize the probability of obtaining new information by introducing a switching mechanism allowing agents to choose between costly rational expectations and costless expectations under sticky information. Thereby, the share of agents with rational expectations becomes endogenous and time-varying. While central results of sticky information models are retained, we find that the share of rational expectations is positively correlated with the variance of the variable forecasted, providing a link to models of near-rationality. Output expectations in our model are generally more rational than inflation expectations, but the share of rational inflation expectations increases with a rising variance of the interest rate. With regard to optimal monetary policy, we find that the Taylor principle provides a necessary and sufficient condition for the determinacy of the model. However, output and inflation stability are optimized if the central bank does not react too strongly to inflation, but rather also targets the output gap with a relatively large coefficient in the Taylor rule.

**Keywords:** Endogenous sticky information, heterogeneous expectations, DSGE models.

**JEL classification:** E31, E52, E61

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# 1 Introduction

The empirical observation that households' inflation and output expectations are not always rational<sup>1</sup>, on the one hand, and that New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models with sticky prices are not able to reproduce the inertia and delayed responses to shocks observed in actual inflation and output data, on the other hand, has triggered several alternative approaches. Among these are the sticky information models by Mankiw and Reis (2001, 2002, 2003, 2007), learning models (e.g. Evans and Honkapohja, 2001, 2003), approaches in behavioral economics and models of near-rationality (e.g. Kahneman and Tversky, 1979, Laibson, 1997 and Akerlof et al., 2000) as well as models with heterogeneous expectations and heuristics such as Branch and McGough (2004) and De Grauwe (2008).

In the sticky information model by Mankiw and Reis (2001, 2002, 2003, 2007) it is assumed that all agents know the relevant model of the economy (in contrast to assuming that agents follow simple heuristics as in, e.g., De Grauwe, 2008), while the costs of acquiring and processing information cause some agents to use old information sets, resulting in so-called sticky information. Thus, only agents obtaining the current information are able to form rational expectations, while those who do not update receive information gradually as news spread through the economy. This type of costless and effortless information acquisition can be thought of as obtained by observing other agents' behavior or by chance, for example through the media, as in the epidemiology model of Carroll (2001, 2003).

Sticky information models in Mankiw and Reis (2001, 2002, 2003, 2007) take the probability of updating to the most recent information set,  $\lambda$ , as an exogenous parameter. Hence, all agents face the same probability of updating their information set and this probability stays constant over time. Building on their approach, we analyze a sticky information DSGE model with heterogeneous expectations and an endogenous and time-varying share of agents with rational expectations,  $\lambda_t$ : Agents of type 1 have rational expectations, while agents of type 2 are subject to sticky information and thus

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<sup>1</sup>See, for example, Thomas (1999) and Mankiw et al. (2003) for a survey.

form expectations using an outdated information set. Since new information is costly due to acquisition and processing costs, in our model agents face a trade-off between the accuracy of their forecasts and a fixed ‘rationality cost’ for obtaining the most up-to-date information.

Introducing a switching mechanism derived in a seminal paper by [Brock and Hommes \(1997\)](#), agents switch to being rational once the losses from sticky information become too high. Thereby, the share of agents that update information each period,  $\lambda_t$ , becomes endogenous and time-varying. Empirical evidence of pervasive heterogeneity in inflation expectations and switching between models of expectation formation is given by [Pfajfar and Zakelj \(2009\)](#) and [Pfajfar and Santoro \(2008\)](#), who find evidence that agents form either rational, sticky or static expectations (analyzing the distribution of the Michigan survey of households for the US) and that they switch models frequently in an experimental setting. In a similar vein, [Maag \(2010\)](#) evaluates heterogeneity in quantitative households’ expectations data from the Swedish Consumer Tendency Survey. Estimating a Gaussian mixture model of underlying distribution densities, the author finds that a large share of households form static expectations based on their perception of actual inflation, while smaller shares form rational, adaptive and static expectations based on official inflation rates, respectively.

Simulating our model, we find considerable time-variation in  $\lambda_t$  once the rationality cost exceeds a certain level. Agents seem to be more rational with respect to output than to inflation expectations, which could be due to the higher coefficient on inflation in the Taylor rule, causing households to confer rationality upon the central bank. Furthermore, the share of rational expectations is positively correlated with the variance of the variable to be forecasted: Agents are willing to pay the cost for up-to-date information if changes in the variable are relatively large and remain inattentive otherwise. This is a central result of models with near-rationality of agents by, e.g., [Akerlof and Yellen \(1985\)](#) and [Akerlof et al. \(1996, 2000\)](#). In addition, we find that the share of rational inflation expectations rises with an increase in the variance of the nominal interest rate: Agents choose to pay more attention to inflation when monetary policy becomes more active. This result is also in

line with near-rationality and emphasizes the strong link between monetary policy and inflation expectations.

With regard to monetary policy, important results of sticky information models in [Mankiw and Reis \(2002, 2007\)](#) are reproduced by our model with endogenous and time-varying  $\lambda_t$ : We also find a hump-shaped response of inflation to a monetary policy shock and that determinacy depends only on the reaction coefficient to inflation in the Taylor rule, as pointed out by [Meyer-Gohde \(2009b\)](#). Analysis of second moments and impulse-response functions shows, however, that optimal monetary policy should not put too much weight on inflation (as long as the Taylor principle is fulfilled). A relatively large weight on the output gap minimizes fluctuations of output in response to monetary policy and cost-push shocks, while inflation is only marginally affected. However, this comes at the risk of overreacting to a positive demand shock, which then produces a small recession in output as monetary policy is tightened overly strictly.

The paper is structured as follows: Section 2 presents a short overview of the related literature on sticky information and models with heterogeneous expectations, section 3 derives the model, while section 4 presents simulation results and policy analysis. Finally, section 5 concludes.

## 2 Literature Review

While DSGE models assume sticky prices with [Calvo \(1983\)](#) pricing, [Mankiw and Reis \(2001, 2002, 2003, 2007\)](#) apply the Calvo mechanism to the arrival of new information, so that agents underly an exogenous probability each period that they will not be able to update their information set. Thereby, although the rational expectations hypothesis is retained, agents are restricted in the sense that new information is distributed slowly throughout the economy. As a result, macroeconomic relations are governed by an infinite sum of lagged expectations, resulting in hump-shaped impulse-responses of inflation and output to shocks and a significantly higher degree of inertia in the

variables.<sup>2</sup> Microfoundations for the sticky information model are derived by Reis (2006a,b), where both consumers and firms rationally choose to only sporadically update their information due to the costs related to acquiring and processing information. However, and although Reis derives the optimal length of inattentiveness for both firms and consumers, in the aggregate the probability of updating is given exogenously for all agents. To our knowledge, so far Branch et al. (2009) present the only attempt to fully endogenize the degree of inattentiveness in a sticky information model. Building on the model in Ball et al. (2005), firms choose their degree of inattentiveness by minimizing a quadratic loss function comparing the firm-specific price given the firm-specific  $\lambda$  to the optimal price given some fixed economy-wide probability of updating information  $\bar{\lambda}$ . The authors furthermore introduce a cost to information gathering that is defined relative to  $\lambda^2$ . However, contrary to our approach the rate of information updating is not time-varying, so that agents solve the optimization problem only once. This is not satisfactory in our view, as agents react to shocks that occur over time.

A different explanation for deviations from rational expectations is given in the models of near-rationality by Akerlof and Yellen (1985) and Akerlof et al. (2000). The authors define near-rationality as "non-maximizing behavior in which the gains from maximizing rather than nonmaximizing are small in a well-defined sense" (Akerlof and Yellen, 1985, pp. 823 - 824), meaning that losses from near-rationality are only second-order in terms of the deviation from the long-run equilibrium, but may nevertheless cause first-order changes in real activity. Building on findings in psychology and behavioral economics that agents are often not fully maximizing agents and employ simplifying mechanisms such as editing, mental framing or heuristics, Akerlof et al. (2000) derive a model of near-rational wage and price setting. In their model, both firms and workers may ignore a fraction (or all) of inflation, if levels of inflation are low, as costs from fully maximizing do not match the gains in profits or wages they produce.

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<sup>2</sup>Several studies have compared models with sticky prices and sticky information. While Trabandt (2007) claims that the hybrid New Keynesian model with sticky prices and habit formation can outperform the sticky information model, Dupor et al. (2006) find evidence for both sticky prices and sticky information in aggregate US data.

A growing literature models deviations from rationality by incorporating heterogeneous expectations into DSGE and also overlapping-generations (OLG) models: Nunes (2009), Guse (2005), Berardi and Duffy (2007) and Berardi (2009) study the impact of non-rational expectations under recursive least-squares learning in the spirit of Evans and Honkapohja (2001, 2003). Analyzing stability of a model where agents either estimate the mean or an AR(1) process of the series to be forecasted, Guse (2005) finds that stability can switch between the fundamental and the AR(1) solution as the level of heterogeneity in expectations varies. While Nunes (2009) incorporates a fraction of learners into an otherwise standard New Keynesian DSGE model to explain the hump-shaped behavior of inflation, Berardi and Duffy (2007) evaluate the role of central bank transparency in a DSGE with private sector learning. Similarly, Berardi (2009) analyzes optimal monetary policy in a model with heterogeneous expectations, where a fraction of agents makes forecasts using an underparameterized model. Comparing equilibria under various Taylor rules, the author finds that stability is not guaranteed if the central bank responds only to learners' expectations using the underparameterized model. Furthermore, welfare outcomes are optimal if monetary policy responds only to rational learners rather than to heterogeneous expectations due to a stronger reaction of rational expectations to shocks in the economy and a more favorable trade-off between the output gap and inflation variability. Woodford (2005) also evaluates optimal monetary policy in the case of near-rational private sector expectations, defined as unspecified deviations from the central bank's expectations. He finds that commitment and history-dependence of optimal policy become even more important with uncertainty about agents' expectations than when assuming rational expectations.

Another strand of the literature explains the effect of heterogeneous expectations when a fraction of agents follows simple heuristics. An earlier contribution is Branch and McGough (2004), who analyze a DSGE model with rational and non-rational expectations, where non-rational agents follow a simple adaptive or trend-extrapolating rule. The authors formulate a heterogeneous expectations equilibrium and find that determinacy and stability of the model depend on the weight of non-rational expectations in

the aggregate expectations operator and on the type of heuristic that non-rational agents follow. [Branch and McGough \(2009\)](#) derive microfoundations for the DSGE with heterogeneous expectations analyzed in [Branch and McGough \(2004\)](#) and give axioms that are necessary in order to be able to derive an aggregate expectations operator in the form of a convex combination of heterogeneous expectations. In a similar model, [Geiger and Sauter \(2009\)](#) evaluate the effect of heterogeneous inflation expectations on optimal monetary policy. In contrast to [Evans and Honkapohja \(2003\)](#), they find that a traditional Taylor rule reacting to actual values of inflation and the output gap works best, as the presence of heterogeneous expectations helps to stabilize the economy so that a forward-looking Taylor rule would only be the best choice if all agents had rational expectations.

Whereas in the studies cited above the proportions of rational and non-rational agents are given exogenously, [Brock and Hommes \(1997\)](#) introduce a mechanism, whereby agents evaluate the performance of their forecast and switch to alternative prediction rules if their expectations deviate too much from current values. The authors use the mechanism to model the choice between rational expectations, that come at a positive cost, and non-rational expectations, that are costless, but imperfect. [Branch and Evans \(2006, 2007, 2009\)](#) use a similar mechanism to the one proposed by [Brock and Hommes \(1997\)](#) to explain switching between predictors in a model where agents are restricted to choose between different underparameterized models to obtain forecasts. In the spirit of learning models such as [Evans and Honkapohja \(2001, 2003\)](#), the authors argue that bounded rationality can lead to misspecification in the form of over-simplistic models, even when agents are rational in the sense that they learn the consistent parameters of the misspecified model. As a result, the authors find that multiple equilibria may exist even when the rational expectations equilibrium would lead to a unique and stable equilibrium. Volatility in output and inflation is generated endogenously in the model, but the authors find that a monetary policy reacting increasingly to households inflation expectations can reduce inflation volatility. A further approach using [Brock and Hommes \(1997\)](#)'s switching mechanism is presented by [De Grauwe \(2008\)](#) who analyzes switching

between simple heuristics regarding output and inflation expectations in a DSGE model without rational expectations. He finds that waves of optimism and pessimism regarding output can generate endogenous business cycles and a proportion of trend-extrapolators of inflation causes endogenous inertia in the simulated inflation series. [Brazier et al. \(2008\)](#) allow for switching between adaptive and rational expectations of inflation in an OLG model and find that monetary policy should account for expected inflation in order to stabilize the economy. Even then, endogenous switching between heuristics causes endogenous volatility in the inflation rate, which can, however, be reduced when an inflation target heuristic is introduced successfully. Similarly, [Brock and de Fontnouvelle \(2000\)](#) analyze an OLG model with heterogeneous inflation expectations and endogenous switching between heuristics. Letting the number of possible predictors tend to infinity, the authors find that the model is asymptotically stable if the mean of the distribution of predictors is low, but becomes unstable for a high mean expected inflation.

### 3 The Model

#### 3.1 Heterogeneous Expectations

We assume there exist two types of agents:

1. Agent 1 has rational expectations:  $E_t^{RE} = E_t$
2. Agent 2 is subject to sticky information:  $E_t^{SI} = \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j}$ ,

Agents of type 2 thus deviate from rational expectations in the sense that they know the relevant model and are computationally able to compute rational forecasts, but do not have access to the most recent information set. Note that agents forecasting with sticky information use outdated information from all past periods up to  $t-1$ , where expectations receive less weight, the further they lie in the past, i.e. the longer a particular agent has not updated his information set. The expectations operator for agents with sticky information,  $E_t^{SI}$ , is hence a weighted aggregate of all agents that use information sets older than the current one. The weighting parameter  $\bar{\lambda}$  can be

interpreted as the average share of agents with rational expectations and is assumed to be constant over time. An agent that decides to pay the rationality cost, will belong to group 1 as long as he pays the cost each period, and becomes an agent of type 2 if he does not update his information set in a certain period.

The aggregate heterogeneous expectations index is then defined as follows:  $\tilde{E}_t \equiv \lambda_t E_t^{RE} + (1 - \lambda_t) E_t^{SI} = \lambda_t E_t + (1 - \lambda_t) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j}$ , where  $\lambda_t$  is the endogenous share of rational agents in period  $t$ , the time-varying analogue to the probability that agents may update their information set, which was given exogenously in the sticky information model. We assume that agents solve the model before deciding between expectations operators. Thus, the model equations can be derived setting  $\lambda_t = \bar{\lambda}$ , as the switching mechanism operates after equilibrium values are found.

Furthermore, we assume that the axioms in [Branch and Evans \(2009\)](#) necessary for aggregation of heterogeneous expectations hold. Thus, standard mathematical operations with expectations operators, like the law of iterated expectations, are assumed to hold also across heterogeneous expectations. Also, second-order interactions between different expectations operators are ruled out, agents are assumed to correctly forecast variables' steady states and in the limit, all agents have rational expectations.

### 3.2 Households' Problem

The model follows a standard New Keynesian set-up, see for instance [Walsh \(2003\)](#) and [Mankiw and Reis \(2007\)](#), but includes heterogeneous expectations  $E_t^j$ : The model economy is populated by a large number of infinitely-lived households, that differ only with respect to the expectations operator used, but otherwise have identical preferences and endowments. Households of type  $j$  maximize utility, where we assume constant relative risk aversion:

$$\max_U U(C_t, N_t) = E_t^j \sum_{k=0}^{\infty} \beta^k \left[ \frac{C_{t+k}^{1-\sigma} - 1}{1-\sigma} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \right] \quad (1)$$

subject to the budget constraint

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t}, \quad (2)$$

(abstracting from money, capital and the government), where the composite consumption good,  $C_t$  and the aggregate price index  $P_t$  are defined as Dixit-Stiglitz aggregators of individual consumption goods  $c_{t,i}$  produced by firm  $i$  and their respective prices:

$$C_t = \left( \int_0^1 c_{t,i}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad (3)$$

$$P_t = \left( \int_0^1 p_{t,i}^{1-\theta} di \right)^{\frac{1}{1-\theta}} \quad (4)$$

From the first-order conditions we get the log-linearized Euler equation:

$$\hat{c}_{t,j} = E_t^j \hat{c}_{t+1,j} - \frac{1}{\sigma} (\hat{i}_t - E_t^j \pi_{t+1}), \quad (5)$$

where variables with a hat denote deviations from steady state. We thus get the standard log-linearized Euler equation for each agent  $j$ , where individual deviations of consumption from its steady state differs according to the expectations operator employed.

Aggregation of the Euler equation in (5) across households makes use of the axioms in [Branch and Evans \(2009\)](#), especially the assumption that the law of iterated expectations holds across heterogeneous expectations and that agents have identical expectations in the limit. Iterating forward and aggregating across agents gives, after some algebra:<sup>3</sup>

$$\hat{c}_t = \tilde{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \tilde{E}_t (\hat{i}_t - \pi_{t+1}), \quad (6)$$

where  $\tilde{E}_t = \lambda_t E_t^{RE} + (1 - \lambda_t) E_t^{SI} = \lambda_t E_t + (1 - \lambda_t) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j}$ . Note that agents under sticky information do not necessarily use the same expectations operator, since the set of agents with sticky information can include any lagged information set up to minus infinity.

<sup>3</sup>For a detailed derivation, see the mathematical appendix.

To arrive at the New Keynesian IS-relation, recall that without investment, government and net exports  $\hat{y}_t = \hat{c}_t + u_t$ ,<sup>4</sup> where we define  $u_t$  as an i.i.d. demand shock. We thus get for the New Keynesian IS curve:

$$\hat{y}_t = \tilde{E}_t \hat{y}_{t+1} - \frac{1}{\sigma} (\hat{i}_t - \tilde{E}_t \pi_{t+1}) + u_t, \quad (7)$$

which gives, when spelling out heterogeneous expectations included in  $\tilde{E}_t$ :

$$\begin{aligned} \hat{y}_t &= \lambda_t \left( E_t \hat{y}_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} \right) + (1 - \lambda_t) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} \left( \hat{y}_{t+1} + \frac{1}{\sigma} \pi_{t+1} \right) \\ &\quad - \frac{1}{\sigma} \hat{i}_t + u_t \end{aligned} \quad (8)$$

As in the standard sticky information model, we thus find that the IS-relation contains an infinite sum of past expectations on steady-state output and inflation, reflecting the fact that a fraction of agents use outdated information sets for forecasting. However, here the fractions of households in each group are determined endogenously by the switching mechanism given below.

### 3.3 Firms' Problem

Next, we model firms' behavior. Again, we assume a large number of firms, that produce individual consumption goods, which together form the composite consumption basket. We also assume that firms are owned by households and are thus subject to the same heterogeneity in expectations faced by households.<sup>5</sup>

In line with the standard New Keynesian model, we assume a Cobb-Douglas production function with constant returns to scale:

$$y_{t,i} = Z_t N_{t,i}, \quad (9)$$

<sup>4</sup>Problems and implications of this approach are discussed in [Groessl \(2008\)](#).

<sup>5</sup>This is in contrast to [Mankiw and Reis \(2007\)](#) who assume that households and firms are subject to different probabilities of updating their information set.

where  $Z_t$  denotes technology (equal for all firms) and  $N_{t,i}$  is the amount of labour used by firm  $i$  to produce  $y_{t,i}$ . Since we assume market clearing, we can set  $Y_t = C_t$ , ignoring government consumption and net exports. Finally, from the cost minimization problem of households, we have for the demand of goods produced by firm  $i$  of household  $j$ :

$$c_{t,j} = y_{t,i} = \left( \frac{p_{t,i}}{P_t} \right)^{-\theta} C_t \quad (10)$$

Again, we will assume that the axioms from [Branch and McGough \(2009\)](#) hold. In line with the framework applied in the sticky information models, we assume that firms set optimal prices each period, hence the only rigidity in the model applies to the stickiness in information. Firms maximize profits subject to the production function in (9) and the demand function in (10):

$$\max_{p_{t,i}} E_t^i \left[ \frac{p_{t,i} y_{t,i}}{P_t} - \frac{W_t N_{t,i}}{P_t} \right] \quad (11)$$

s.t.

$$y_{t,i} = Z_t N_{t,i} \Rightarrow N_{t,i} = y_{t,i} Z_t^{-1} \quad (12)$$

$$y_{t,i} = \left( \frac{p_{t,i}}{P_t} \right)^{-\theta} C_t \quad (13)$$

Inserting the constraints and log-linearizing gives for the deviation of the optimal price set by firm  $i$  from its steady state:

$$\hat{p}_{t,i}^* = E_t^i [\hat{p}_t + \hat{\varphi}_t] \quad (14)$$

where  $\hat{\varphi}_t$  is the deviation from steady state of real marginal costs defined as  $\varphi_t \equiv (W_t/P_t)Z_t^{-1}$ . In order to express the optimal price in terms of the output gap rather than real marginal costs, we follow [Ball et al. \(2005\)](#) and get for the aggregate price index:<sup>6</sup>

<sup>6</sup>For a detailed derivation, see the mathematical appendix.

$$\begin{aligned}
\hat{p}_t &= \lambda_t E_t^{RE} [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t] + (1 - \lambda_t) E_t^{SI} [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t] \\
&= \tilde{E}_t [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t]
\end{aligned} \tag{15}$$

where  $\psi = \frac{\sigma + \eta}{1 + \eta \theta}$ ,  $\hat{y}^n$  is the natural output under flexible prices and full information and  $e_t$  is an i.i.d. cost-push shock. Finally, lagging equation (15) by one period and subtracting it from (15), setting  $\lambda_t = \bar{\lambda}$ , gives for the sticky information Phillips curve:<sup>7</sup>

$$\begin{aligned}
\pi_t &= \frac{\psi \bar{\lambda}}{1 - \bar{\lambda}} (\hat{y}_t - \hat{y}_t^n) + \frac{\bar{\lambda}}{1 - \bar{\lambda}} e_t \\
&\quad + \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} [\pi_t + \psi \Delta (\hat{y}_t - \hat{y}_t^n) + \Delta e_t]
\end{aligned} \tag{16}$$

In order to derive the sticky-information Phillips curve with heterogeneous expectations, we then allow agents to choose between expectations operators in period t-1, introducing the time-varying share of rational agents  $\lambda_{t-1}$ :

$$\begin{aligned}
\pi_t &= \frac{\psi \bar{\lambda}}{1 - \bar{\lambda}} (\hat{y}_t - \hat{y}_t^n) + \frac{\bar{\lambda}}{1 - \bar{\lambda}} e_t + \lambda_{t-1} E_{t-1} [\pi_t + \psi \Delta (\hat{y}_t - \hat{y}_t^n) + \Delta e_t] \\
&\quad + (1 - \lambda_{t-1}) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-2-j} [\pi_t + \psi \Delta (\hat{y}_t - \hat{y}_t^n) + \Delta e_t]
\end{aligned} \tag{17}$$

Introducing the time-varying share of rational agents,  $\lambda_{t-1}$ , hence causes inflation rates to be influenced by a moving average of past shares of rational, i.e. inattentive, agents. As before, when inserting heterogeneous expectation formation, we see that due to the fraction of agents under sticky information,

<sup>7</sup>For a detailed derivation, see the mathematical appendix.

an infinite sum of past expectations features also in the Phillips curve with flexible prices.

### 3.4 Monetary Policy Rule

As usual in the New Keynesian DSGE framework, our model is closed by defining a monetary policy rule. In line with the sticky information DSGE model in [Mankiw and Reis \(2007\)](#), we assume a Taylor rule with interest rate smoothing, targeting actual values of inflation and the output gap:

$$\hat{i}_t = \mu_i \hat{i}_{t-1} + (1 - \mu_i) (\mu_\pi \pi_t + \mu_{y^{gap}} (\hat{y}_t - \hat{y}_t^n)) + \eta_t, \quad (18)$$

where  $\eta_t$  is an i.i.d. shock to monetary policy. Note that we assume the central bank to be rational with respect to current values of inflation and the output gap, while at the same time accounting for the heterogeneity in agents' expectations implicitly contained in realized values of inflation and the output gap.

### 3.5 Switching Mechanism

Finally, we introduce an endogenous switching mechanism, which allows agents to choose between full and sticky information. Note that the mechanism applies to both households and firms, as we assume that firms are owned by households, and that it governs the expectations of both inflation and excess demand. We follow the mechanism developed by [Brock and Hommes \(1997\)](#) that has also been employed, for instance, in [De Grauwe \(2008\)](#) and [Brazier et al. \(2008\)](#).<sup>8</sup>

Agents are confronted with a choice problem of the following kind: On the one hand, they face a positive cost of acquiring and processing information necessary in order to form rational forecasts, which we define as 'rationality cost'. On the other hand, they have the prospect of gaining in consump-

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<sup>8</sup>Note that we do not face a problem of chaotic dynamics and multiple equilibria here, as in [Brock and Hommes \(1997\)](#), since we assume that the model is solved first and agents consequently decide between models of expectations. This rules out feedback effects from agents' switching behavior to the equilibrium.

tion and profits if the rational forecast produces a more accurate solution to the utility and profit optimization problems than forecasts under sticky information. The reverse argument applies to choices regarding sticky information, where we assume that there is no cost of obtaining past information. Agents' choice problem thus relates to the literature on 'rational inattention' founded by [Sims \(2003\)](#), which analyzes rational deviations from full information due to limited information processing capacities. However, in contrast to [Sims \(2003\)](#) we assume that it is possible for agents to attain rationality once they are willing to pay the cost for it. Deriving microfoundations for the sticky information model, [Reis \(2006a,b\)](#) computes the optimal length of inattentiveness for households and firms and finds that it falls with the volatility of income shocks and the difference between profits under full or limited information, but increases with the costs of updating consumption and production plans. In that sense, the microeconomic sticky information model already incorporates the choice problem introduced here. However, it plays no role in the macroeconomic model, where it is assumed that all agents face the same exogenous probability of updating their information set each period, modelled as a kind of Calvo mechanism.

Applying aspects of discrete choice theory to analyze how agents choose between rational and sticky information expectations, we assume that agents continuously evaluate their past forecast performance against current data. As in [Brock and Hommes \(1997\)](#), agents measure the accuracy of their expectations operator by computing past mean squared forecast errors:

$$U_t^{RE} = - \sum_{k=0}^{\infty} [\omega_k (\hat{x}_{t-k} - E_{t-k-1} \hat{x}_{t-k})^2 + K^{RE}] \quad (19)$$

$$U_t^{SI} = - \sum_{k=0}^{\infty} \left[ \omega_k (\hat{x}_{t-k} - \bar{\lambda} \sum_{j=k-1}^{\infty} (1 - \bar{\lambda})^j E_{t-j-1} \hat{x}_{t-k})^2 \right], \quad (20)$$

where equation (19) gives forecast performance of rational expectations and (20) that of expectations under sticky information, respectively. Each period, agents evaluate the forecast performance of their current expectations operator against the realizations of the forecasted variable in that period,

which recursively adds up to the sum of all forecast errors. Note that only rational agents face the positive rationality cost  $K^{RE}$  of obtaining up-to-date information. The weights  $\omega_k$  are assumed to be geometrically declining and sum to one, defined as  $\omega_k = (1 - \rho)\rho^k$ , with  $0 < \rho < 1$  measuring the degree of agents' memory of past mean squared forecast errors.

Solving backwards we get, after some algebra, for the forecast performance of rational expectations:<sup>9</sup>

$$U_t^{RE} = \rho U_{t-1}^{RE} - (1 - \rho) [(\hat{x}_t - E_{t-1}\hat{x}_t)^2 + K^{RE}] \quad (21)$$

Similarly, we get for expectations under sticky information:

$$U_t^{SI} = \rho U_{t-1}^{SI} - (1 - \rho) (\hat{x}_t - \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-j-2}\hat{x}_t)^2 \quad (22)$$

It can thus be seen that this period's forecast performance of a particular process for expectation formation is a weighted average of its squared forecast error this period and last period's forecast performance, incorporating mean squared forecast errors of previous periods. As  $\rho$  approaches zero in the limit, agents' memory becomes shorter and the forecast performance is solely based on this period's squared forecast error. Conversely, as  $\rho$  converges towards one, agents put more weight on the forecast performance of previous periods and tend to ignore the most recent squared forecast error.

Finally, following [Brock and Hommes \(1997\)](#), the probability that agents choose to be rational, i.e. the share of rational agents, is defined each period as follows:

$$\lambda_t = \frac{\exp(\gamma U_t^{RE})}{\exp(\gamma U_t^{RE}) + \exp(\gamma U_t^{SI})}. \quad (23)$$

Consequently, the probability that agents forecast with sticky information equals:

$$(1 - \lambda_t) = \frac{\exp(\gamma U_t^{SI})}{\exp(\gamma U_t^{RE}) + \exp(\gamma U_t^{SI})}, \quad (24)$$

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<sup>9</sup>For a detailed derivation, see the mathematical appendix.

where the parameter  $\gamma$  measures the so-called ‘intensity of choice’, that is the degree to which agents let their choice of an expectations rule be influenced by its past forecasting performance. We thus model the proportion of rational agents as a function of the relative utility from being rational compared to overall utility levels under both rational and sticky information. As long as the rationality costs outweigh the gains from the relatively higher accuracy of rational forecasts, more agents will choose to forecast with old information. But, as rationality becomes more important, for example due to higher levels or a higher volatility of inflation, the gains from rationality increase and more agents will choose to be fully rational.

Since in our model agents form expectations both on future inflation and future output (i.e. the output gap), we need to define two separate switching mechanisms, where agents evaluate the accuracy of the inflation and output forecasts under rational and sticky information, respectively, so that  $\hat{x}_t$  in equations (21) and (22) is replaced with  $\hat{y}_t$  and  $\pi_t$ , respectively. This gives for the time-varying shares of agents with rational output and inflation expectations:

$$\lambda_t^y = \frac{\exp(\gamma U_{y,t}^{RE})}{\exp(\gamma U_{y,t}^{RE}) + \exp(\gamma U_{y,t}^{SI})}. \quad (25)$$

$$\lambda_t^\pi = \frac{\exp(\gamma U_{\pi,t}^{RE})}{\exp(\gamma U_{\pi,t}^{RE}) + \exp(\gamma U_{\pi,t}^{SI})}. \quad (26)$$

Distinguishing between heterogeneous expectations regarding output and inflation, we then obtain for the IS curve and the Phillips curve:

$$\begin{aligned} \hat{y}_t &= \lambda_t^y E_t \hat{y}_{t+1} + \frac{1}{\sigma} \lambda_t^\pi E_t \pi_{t+1} + (1 - \lambda_t^y) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} (\hat{y}_{t+1}) \\ &+ \frac{1}{\sigma} (1 - \lambda_t^\pi) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} (\pi_{t+1}) - \frac{1}{\sigma} \hat{i}_t + u_t \end{aligned} \quad (27)$$

$$\begin{aligned}
\pi_t &= \frac{\psi\bar{\lambda}}{1-\bar{\lambda}}(\hat{y}_t - \hat{y}_t^n) + \frac{\bar{\lambda}}{1-\bar{\lambda}}e_t \\
&+ \lambda_{t-1}^y \psi E_{t-1} (\Delta(\hat{y}_t - \hat{y}_t^n) + \Delta e_t) + \lambda_{t-1}^\pi E_{t-1} \pi_t \\
&+ (1 - \lambda_{t-1}^\pi) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-2-j} \pi_t \\
&+ (1 - \lambda_{t-1}^y) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-2-j} (\psi \Delta(\hat{y}_t - \hat{y}_t^n) + \Delta e_t)
\end{aligned} \tag{28}$$

## 4 Results

### 4.1 Model Simulation

The model is solved numerically using the algorithm from [Meyer-Gohde \(2009a\)](#), which accounts for the infinite sum of lagged expectations included in sticky information models by calculating matrices of limiting coefficients as the sum approaches minus infinity. Simulations are carried out over 1500 periods, where the first 500 periods are used to initialize the model and generate lagged expectations and are then dropped. Calibrated parameters are taken from [McCallum \(2001\)](#) for the baseline model and shown in [Table A1](#) in the appendix. The parameters  $\eta$  and  $\theta$  are not included in [McCallum \(2001\)](#)'s model, we set values to obtain a value for the elasticity of inflation with respect to the output gap similar to the one estimated by [Mankiw and Reis \(2007\)](#). Furthermore and in line with [De Grauwe \(2008\)](#), we set the standard deviation of the demand and cost-push shocks in the IS and Phillips curve equal to each other.<sup>10</sup>

<sup>10</sup>Of course there exist numerous calibration approaches to DSGE models such as ours. As a robustness check, we compared impulse-response functions and simulated series for the share of rational agents from our baseline model to models with parameters from [Mankiw and Reis \(2007\)](#) and [De Grauwe \(2008\)](#). Allowing for autocorrelation of the shocks as in [Mankiw and Reis \(2007\)](#) or for lagged endogenous variables as in [De Grauwe \(2008\)](#) considerably increases the persistence of the simulated series and the impulse-responses. Otherwise, our results remain robust. The simulations with alternative calibrations are

The switching parameter  $\gamma$  in equations (25) and (26) is set to 10000 as in Brock and Hommes (1997) and agents' degree of memory of past forecast errors  $\rho$  in equations (21) and (22) is taken to be 0.5 as in De Grauwe (2008).<sup>11</sup> We set initial values of  $\lambda_t$  for both inflation and output expectations to 0.5. The rationality cost is defined relative to the mean squared forecast error of expectations under sticky information. Simulating the model over 1000 periods, assuming no costs of rationality and no persistence in forecast errors, we obtain mean values of  $U_{y,t}^{SI}$  and  $U_{\pi,t}^{SI}$ . Parameters of  $K_y^{RE}$  and  $K_\pi^{RE}$  then range from 0% - 100% of the mean forecast error under sticky information, see Table (A2) in the Appendix. We take the value of  $K_{y/\pi,t}^{RE} = 50\%$  of  $U_{y/\pi,t}^{SI}$  as the baseline cost.

## 4.2 Simulation Results and Impulse-Response Functions

Second moments and first-order serial correlation coefficients of inflation, output and nominal interest rate series simulated with the McCallum (2001) calibration are presented in Table 1 for varying rationality costs. All values were obtained by simulating the model 1000 times over 1000 periods.

While the standard variation of the simulated series generally does not vary with increasing rationality costs, the degree of persistence in the series seems to increase with higher costs (and decreases again slightly at costs of  $K^{RE} = 100\%$  for  $\pi_t$  and  $\hat{i}_t$ ). This is in line with the intuition that as rationality costs rise, a higher share of agents will opt for using outdated information for their forecast, thereby increasing the endogenous persistence of output and inflation, and consequently the interest rate. Nevertheless, the persistence of the simulated series for output and inflation cannot match the high degree of autocorrelation found in empirical data. In the case of inflation, the model even suggests a negative correlation coefficient. This problem often encountered in standard DSGE models is due to the fact that no au-

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available from the author upon request.

<sup>11</sup>We checked for robustness of our results with values of  $\gamma$  and  $\rho$  ranging between  $0 \leq \gamma \leq 15000$  and  $0 \leq \rho \leq 1$ . As expected, switching becomes more frequent, the higher the value of  $\gamma$  and the lower the value of  $\rho$ , respectively. Nevertheless, overall our results remain robust also for high and low values of both  $\gamma$  and  $\rho$ . Results are available from the author upon request.

to correlation in the shocks was assumed as, e.g. in the sticky information DSGE of [Mankiw and Reis \(2007\)](#) and, apart from the coefficient of interest rate smoothing, no lagged endogenous variables were included as in, e.g. the [De Grauwe \(2008\)](#) model. Hence, sticky information by itself is not able to produce the inertia necessary to reproduce empirical properties of inflation and output data.

< *Table 1 here* >

Figures 1 - 4 show impulse-response functions of inflation, output and the nominal interest rate to a one-standard-deviation demand shock in the IS curve, a cost-push shock in the Phillips curve, a monetary policy shock and a technology shock. As expected, the interest rate increases after a positive demand shock and then returns slowly to its steady state. Inflation also increases after a demand shock and undershoots before returning to its steady state value, which is due to the fact that firms target prices and not inflation.

< *Figure 1 here* >

A positive cost-push shock causes interest rates to rise while simultaneously dampening output, leading to a persistent recession and negative deviations of inflation from its steady state from the second quarter after the shock onwards.

< *Figure 2 here* >

The impulse-response functions of a one-standard-deviation rise in the interest rate shown in Figure 3 show the typical hump-shaped response of inflation obtained from sticky information Phillips curves, e.g. in [Mankiw and Reis \(2002, 2006, 2007\)](#): A positive shock to the nominal interest rate has a gradually dampening effect on inflation, which continues to hold until two years after the shock, when all series have returned to their steady states. Hence, one important feature of the sticky information models is retained also with heterogeneous expectations. The impulse-response function of output

to a monetary shock also shows considerable persistence, albeit without a hump-shaped pattern. This is due to the fact that our IS curve deviates from the one derived in [Mankiw and Reis \(2007\)](#) in that it includes past expectations of future, instead of present, inflation and output.

*< Figure 3 here >*

Finally, a positive technology shock has a pronounced boosting effect on output, which is due to the autocorrelation assumed in the shock. Both inflation and the interest rate are reduced in response to the shock, leading to the long-lasting boom in output.

*< Figure 4 here >*

### 4.3 Time-varying Shares of Rational Expectations

#### 4.3.1 Rational Expectations with Varying Rationality Costs

After evaluating the dynamics of the simulated variables in the model, we turn to analyzing the dynamics of the time-varying shares of rational inflation and output expectations. From [Table 2](#) we see that the shares of agents with rational output expectations,  $\lambda_t^y$ , and with rational inflation expectations,  $\lambda_t^\pi$ , only fluctuate between zero and one if the rationality cost is at least 25% of the mean squared forecast error under sticky information. Even then, on average as much as 80% and 73%, respectively, of all agents have rational output and inflation expectations. Hence, even when the costs for new information are relatively high, a large proportion of agents will prefer to be rational. As the rationality cost increases to 50% of the mean squared forecast error and higher, on average the share of rational agents approaches zero. Nevertheless, even if new information is costless, so that on average close to every agent will be rational, there may occur shocks such that between one third and one half of the population chooses to ignore the new information, at least for short periods of time.

*< Table 2 here >*

With regard to the standard deviation of the shares of rational agents, for both output and inflation expectations variability is highest at medium costs of 50%. This implies that at this cost, agents have no predetermined preference for either rational expectations or those under sticky information, but rather switch to their preferred expectations operator depending on current shocks to the economy. At lower costs, the advantages of rational expectations generally outweigh the costs. Conversely, at higher costs the gains in utility and profits from rational forecasts can generally not make up for the high costs of new information.

Interestingly, whereas the variability between rational and sticky expectations is approximately equal for output and inflation expectations, on average agents seem to be more rational with respect to output expectations than to inflation expectations: At costs of 50% of the mean squared forecast error, on average 50% of agents choose to forecast with rational expectations regarding output, but only 34% form rational inflation expectations. The difference is highest at medium costs and decreases with costs falling to 0% or rising to 100%. It thus seems that agents are more concerned about obtaining new information on changes in output than on inflation. This could be explained by the relatively larger weight of inflation compared to the output gap in the central bank's Taylor rule.<sup>12</sup> Hence, if monetary policy convincingly targets inflation, agents feel that they can afford to delegate rationality to the central bank by paying less attention to current shocks on inflation.<sup>13</sup>

Furthermore, we analyze the average cycle length of  $\lambda_t^y$  and  $\lambda_t^\pi$ , that is the average number of periods in which agents use their forecasting rule

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<sup>12</sup>Our finding is robust also with a high weight on the output gap relative to inflation in the Taylor rule: As monetary policy puts a larger weight on the output gap, mean values of  $\lambda_t^y$  and  $\lambda_t^\pi$  converge, and *vice versa*. Nevertheless, in order to guarantee determinacy, the central bank has to react more than one-to-one to changes in inflation, which might be enough for agents to concentrate more on information regarding recent developments in output.

<sup>13</sup>For example, [Bryan and Palmqvist \(2005\)](#) analyze survey data of households' inflation expectations for Sweden and the US, and report that the introduction of the Swedish inflation target of 2% significantly increased the proportion of Swedish households who ignore inflation in the recent period of low inflation rates. Conversely, even though inflation was very similar in the US, this effect is not evident in American households' inflation expectations, suggesting an important role of central bank communication for the formation of inflation expectations.

before switching to an alternative rule. We define cycles of rationality and non-rationality, respectively, where simulated values of  $\lambda_t$  belong to the same cycle if they deviate from the previous period's value by not more than a tolerance parameter of 0.001. For both  $\lambda_t^y$  and  $\lambda_t^\pi$  switching is most frequent at a medium cost of 50%, where on average agents switch rules for output forecasting every three quarters and for inflation forecasting every four quarters. Generally, the switching frequency increases as the rationality cost rises from 0 to 50%, and then decreases again as it further rises to 100%. Especially at very high costs of 100% of the mean squared forecast error agents seem very reluctant to switch forecasting rules and keep rules for an average of 11 and 21 quarters, respectively. Comparing the shares of agents with rational output and inflation expectations, our model suggests that for rationality costs of 50% and higher there is considerably more switching of forecasting rules for output than for inflation, while at lower costs switching frequencies are relatively similar. Again, this indicates a possible link between inflation targeting by the central bank and the rationality of inflation expectations: At a noticeable cost for rational expectations, agents feel a stronger need to adjust output expectations and switch inflation forecasting rules less frequently, since they know via the Taylor rule that the central bank will pay more attention to stabilizing inflation than output.

#### 4.3.2 Rational Expectations and the Volatility of Inflation and Output

After analyzing the time-varying shares of rational and sticky expectations with increasing rationality costs, it remains to evaluate which macroeconomic conditions foster rational expectations. There exists a large literature on the question of disagreement among forecasters and its relation to the level and the volatility of inflation and output. To mention just a few, [Mankiw et al. \(2003\)](#) analyze disagreement in inflation expectations for the US and find that the sticky information model is well suited to explain the degree of heterogeneity in forecasts. Similarly, [Carroll \(2001, 2003\)](#) proposes micro-foundations for the sticky information model in an 'epidemiology model' for

inflation expectations, where agents obtain news on inflation via the media. The speed of arrival of news relates to both the level and the volatility of inflation, since we can assume a higher media coverage on inflation in times of high inflation rates or shocks to inflation. While Carroll tests his model empirically for households' inflation expectations in the US, [Maag and Lamla \(2009\)](#) investigate data for Germany and find supportive evidence of Carroll's model. [Capistran and Timmermann \(2009\)](#) also find that heterogeneity in inflation expectations differs systematically with both the level and the variance of inflation. With respect to firms' expectations regarding the future business outlook, [Lamla et al. \(2007\)](#) provide evidence that firms react strongly to aggregate news shocks, where the impact of the shock differs across sectors.

For the baseline model with a rationality cost of 50% of the mean squared forecast error under sticky information, we analyze the relation between the time-varying share of rational agents and the time-varying variance in the variable to be forecasted. The time-varying volatility of inflation or output is defined as a five-month moving-average of their variance.

*< Table 3 here >*

Table 3 summarizes correlation coefficients of  $\lambda_t^y$  and  $\lambda_t^\pi$  with respect to the level and variance of all endogenous macroeconomic variables of our model. We find that the share of agents with rational expectations is positively correlated with the variance of the variable forecasted, while correlations with the level of the forecasted variable are low. This suggests that agents choose to be more attentive with respect to inflation and output, as volatility, and hence uncertainty, with respect to the variable increases. Conversely, if inflation and output are relatively stable, agents can afford to ignore new information, since losses from forecasting with outdated information will be low.

The share of rational output expectations  $\lambda_t^y$  does not seem to be significantly correlated with any macroeconomic variable other than the variance in output. Conversely, the share of rational inflation expectations  $\lambda_t^\pi$  is also correlated to the monetary policy stance: In addition to the variance of the

inflation rate, the variance of the nominal interest rate seems to play an important role, suggesting that more agents choose to pay the cost for rational inflation expectations as monetary policy becomes more active.

Our results provide a link between models of near-rationality by, e.g., [Akerlof and Yellen \(1985\)](#) and [Akerlof et al. \(2000\)](#), and models of sticky information: With near-rationality, it is assumed that agents will ignore a fraction of inflation as they set wages (and prices) as long as inflation remains below a certain threshold, resulting in a long-run trade-off between inflation and output. However, near-rational models take the threshold of inflation to be given and do not give any microfoundations with regard to agents' behavior when not forming rational expectations. By contrast, in our model agents are optimizing in the sense that they choose the optimal expectations formation process each period. Furthermore, we model expectations under sticky information explicitly as the alternative to rational expectations.

#### 4.4 The Role of Monetary Policy

In this section, we analyze monetary policy under endogenous sticky information. Specifically, determinacy and optimal monetary policy are evaluated across a range of reaction coefficients  $\mu_\pi$  and  $\mu_{ygap}$ .

Since in the limit all agents are assumed to have rational expectations, the well-known eigenvalue accounting method developed by [Blanchard and Kahn \(1980\)](#) can be used to evaluate determinacy also in our model with heterogeneous expectations. We solve the model with six endogenous variables numerically, using the baseline calibration given above for Taylor-rule coefficients on inflation and the output gap ranging between zero and two:<sup>14</sup>  $0 \leq \mu_\pi \leq 2$  and  $0 \leq \mu_{ygap} \leq 2$ .

In contrast to standard determinacy results for DSGE models, found e.g. in [Woodford \(2003\)](#), we find that stability of our model depends only on  $\mu_\pi$ : As long as the Taylor principle is fulfilled and the central bank reacts more

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<sup>14</sup>The upper-bound value of two for  $\mu_\pi$  and  $\mu_{ygap}$  is chosen somewhat ad hoc. It seems to be a reasonable boundary, however, as it gives equal weights to the regions below and above the Taylor principle of  $\mu_\pi \geq 1$ . Furthermore, our results are robust to extending the upper bound to higher values.

than one-to-one to an increase in the inflation rate, the model will have a unique and stable solution. We thus find no long-run trade-off between inflation and the output gap. This result has been confirmed analytically by Meyer-Gohde (2009b) for a DSGE model with a sticky information Phillips curve and is due to the assumption that in the infinite horizon, the model converges to the perfect foresight model with a vertical long-run Phillips curve. As a result, the Taylor principle becomes a necessary and sufficient condition for determinacy. Only at  $\mu_\pi \equiv 1$ , stability of the model is established by the coefficient on the output gap  $\mu_{y^{gap}}$ , however, no clear pattern emerges. Our result regarding determinacy is robust across all values of  $0 < \lambda^{y,\pi} < 1$ . By contrast, Branch and McGough (2009) find for a DSGE with heterogeneous expectations that the share of non-rational expectations influences stability, either positively if non-rational expectations are adaptive, or negatively if non-rational expectations are extrapolative.

Next, we analyze optimal monetary policy for all values of  $\mu_\pi$  and  $\mu_{y^{gap}}$  that lead to a stable equilibrium, i.e.  $1 < \mu_\pi \leq 2$  and  $0 \leq \mu_{y^{gap}} \leq 2$ .<sup>15</sup> Figure 5 shows standard deviations of inflation, output and the interest rate across values of  $\mu_\pi$  and  $\mu_{y^{gap}}$ , where we simulated the model 1000 times over 1000 periods to gain robust results.

< Figure 5 here >

Regarding the central bank's attentiveness towards inflation, the well-known short-run trade-off between inflation and output emerges:<sup>16</sup> Increasing the reaction coefficient to inflation in the Taylor rule,  $\mu_\pi$ , will stabilize inflation at the cost of increasing variability in output. However, in absolute terms our model suggests that the increase in the standard deviation of output is almost five times the decrease in the standard deviation of inflation. Also, variability in nominal interest rates increases with rising  $\mu_\pi$  as monetary policy needs to react more forcefully to changes in inflation.

<sup>15</sup>Simulation results were obtained with starting values of  $\lambda_t^{y,\pi} = 0.5$ . While one coefficient was varied, the other was set equal to its baseline calibration value.

<sup>16</sup>In a DSGE model with a sticky-information Phillips curve, the exact size of the short-run trade-off in period 0 is  $\pi_0 = \frac{\psi\lambda_0}{1-\lambda_0}(\hat{y}_0 - \hat{y}_0^n)$ , see Meyer-Gohde (2009b).

With respect to the central bank's reaction coefficient to the output gap,  $\mu_{ygap}$ , a different picture emerges: Varying the coefficient between  $0 \leq \mu_{ygap} \leq 2$  seems to have no clear effect on the variability of inflation. By contrast, output is stabilized considerably if the central bank reacts strongly to changes in the output gap. Interestingly, variability of the nominal interest rate is minimized at a value of  $\mu_{ygap} = 0.5$ , which is the value assumed in the [McCallum \(2001\)](#) baseline calibration.

Overall, it seems that even though  $\mu_{ygap}$  is not important for determinacy of the model, it is nevertheless relevant for a higher welfare in terms of less output variability. While  $\mu_{\pi} > 1$  can only marginally reduce variability of inflation, output can be stabilized considerably by both keeping  $\mu_{ygap}$  relatively high and  $\mu_{\pi}$  relatively low (providing the Taylor principle is fulfilled). This finding is in line with [De Grauwe \(2008\)](#), who reports that the central bank can to some extent stabilize both inflation and output relative to the case with strict inflation targeting.

< *Figure 6 here* >

< *Figure 7 here* >

Finally, we compare impulse-responses obtained with the lower- and upper-bound values of  $\mu_{\pi}$  and  $\mu_{ygap}$ , setting the other parameter to its baseline calibration value, respectively. From [Figure 6](#) we see that a higher coefficient on inflation in the Taylor rule will stabilize both output and inflation after a monetary policy shock, while the hump-shaped response of inflation is retained. However, this effect comes at the cost of a more severe recession in output after a cost-push shock, as interest rates are forced to increase more forcefully after a shock to inflation occurs. Hence, the short-run trade-off between stabilizing inflation and stabilizing output, analyzed, e.g. by [Geiger and Sauter \(2009\)](#), becomes evident also in our model with endogenous sticky information.

Comparing impulse responses with  $\mu_{ygap} = 0.01$  and  $\mu_{ygap} = 2$  in [Figure 7](#), again inflation and output are stabilized considerably in response to a monetary policy shock, if the central bank responds with a higher coefficient

to changes in the output gap. In contrast to the result in Figure 6, the recession in output after a cost-push shock is also mitigated with a higher  $\mu_{ygap}$  in the Taylor rule, confirming our welfare result from Figure 5. However, a stronger reaction of the central bank to the output gap leads to a more pronounced increase of the nominal interest rate after a positive demand shock, causing output to undershoot so that a recession occurs after the shock. Hence, a high reaction coefficient on the output gap in the Taylor rule seems well suited to stabilize output and reduce recessions after a positive cost-push shock to inflation, but may cause a small recession after a positive demand shock due to an overly strong increase of the nominal interest rate.

## 5 Conclusion

We present a sticky information DSGE model where agents can choose each period between rational expectations and expectations under sticky information: Assuming that all agents know the relevant model, rational expectations produce perfect forecasts, but the new information set can only be obtained at a positive cost, the rationality cost. By contrast, outdated information under sticky information reaches agents freely, but may lead to biased forecasts. Employing a switching mechanism by [Brock and Hommes \(1997\)](#), we thus derive a sticky information DSGE where the share of agents with rational expectations is endogenous and time-varying.

Results from numerical simulation suggest that the share of agents with rational expectations varies between zero and one if the rationality cost corresponds to at least 50% of the mean squared forecast error under sticky information. However, even at zero costs, agents switch between forecasting rules as shocks hit the economy, emphasizing the relevance of a time-varying  $\lambda_t$ .

An important result of our model is the link it provides between models of sticky information and models of near-rationality à la [Akerlof and Yellen \(1985\)](#) and [Akerlof et al. \(2000\)](#): We find that the share of rational expectations is positively correlated with the variance of the variable to be forecasted. Hence, as in the models with near-rationality, agents form rational expecta-

tions if variability in the economy increases and can afford to use outdated information if variables remain relatively stable. The relevance of monetary policy for inflation expectations is highlighted by the result that the share of agents with rational inflation expectations is additionally positively correlated with the variance of the interest rate. Hence, a more active monetary policy is interpreted as a signal to pay closer attention to inflation. This corresponds to our finding that in general agents are more rational with respect to output than with respect to inflation expectations and thus rely on the central bank to maintain a stable inflation rate.

Our model with endogenous  $\lambda_t$  preserves important results of the sticky information models in [Mankiw and Reis \(2002, 2007\)](#): We find that inflation has a hump-shaped response to a monetary policy shock, implying that the maximum impact of the shock occurs with a delay of some periods. This result is obtained even though we do not assume any autocorrelation in the shocks. Furthermore, we can confirm the finding by [Meyer-Gohde \(2009b\)](#) that determinacy in sticky information models depends only on the Taylor rule coefficient of inflation and the model has a unique and stable equilibrium as long as the Taylor principle is fulfilled. With regard to the high persistence in simulated series for inflation and output, we can only reproduce results of [Mankiw and Reis \(2002, 2007\)](#) if we assume autocorrelated shocks like in [Mankiw and Reis \(2007\)](#). Hence, sticky information does not suffice to generate the degree of inertia observed empirically. Nevertheless, the persistence of simulated variables generally increases as the rationality cost rises and more agents choose sticky information.

Analyzing optimal monetary policy in our DSGE with endogenous sticky information, we find that although the Taylor principle is a necessary and sufficient condition for determinacy, welfare analysis shows that the central bank should also target the output gap: Output is stabilized best with a high coefficient on the output gap in the Taylor rule, which does not affect the inflation rate negatively. As long as the Taylor principle is fulfilled, varying the coefficients in the Taylor rule has only marginal effects on inflation stability, but increasing (decreasing)  $\mu_{y^{gap}}$  ( $\mu_{\pi}$ ) will stabilize output. Furthermore, a stronger reaction of monetary policy to inflation will reduce the effect of

a monetary shock on inflation (and, to a smaller degree, also output), but increases the recession after a cost-push shock to inflation. By contrast, a higher coefficient on the output gap will stabilize both output and inflation after monetary policy shocks and reduces the negative response of output after a cost-push shock. We thus conclude that optimal monetary policy should react more than one-to-one to changes in inflation, but nevertheless should not put too much weight on inflation relative to the output gap due to the short-run trade-off between inflation and the output gap. Thus, in terms of welfare our model suggests a role for output targeting in addition to an inflation target. Nonetheless, stabilizing inflation remains an important task for monetary policy, due to the strong link between heterogeneity in inflation expectations and monetary policy actions in our model.

Notwithstanding our results, there may exist further reasons for a relatively high coefficient on inflation in the Taylor rule, such as the prevention of time-inconsistency of monetary policy due to insufficient commitment, causing an inflationary bias. Here, we assume that the central bank always follows the mechanism given by the Taylor rule, such that problems of time-inconsistency cannot arise. Nevertheless, the analysis of our model under commitment and discretion of monetary policy would be an interesting aspect, which we leave for further research.

While to the best of our knowledge this is the first approach allowing agents to choose between rational and sticky information expectations, this paper stands in the tradition of models with heterogeneous expectations, where agents generally choose between rational and non-rational expectations: For instance, [De Grauwe \(2008\)](#) finds that a fraction of agents forecasting with simple heuristic may cause endogenous inertia in inflation and output and that the central bank can reduce variability in both inflation and output when it targets output in addition to inflation. Similarly, [Brazier et al. \(2008\)](#) and [Branch and Evans \(2006, 2007, 2009\)](#) in their models with heterogeneous expectations also report a strong link between heterogeneous inflation expectations and monetary policy in the sense that heterogeneity in expectations may cause endogenous volatility in inflation, which in turn is mitigated once monetary policy targets inflation. However, a trade-off

between inflation and output targeting is generally not analyzed in these models. In line with these models, we also find that the shares of rational agents react to changes in the volatility of the series forecasted. However, we are not able to reproduce feedback effects from the switching of forecasting rules to the model variables, since the switching mechanism operates after the model solution is found. An interesting avenue for future research would be to incorporate the switching mechanism into the solution algorithm by Meyer-Gohde (2009a), accounting for the infinite sum of past expectations present in the sticky information expectations operator.

## 6 Appendix

### 6.1 Mathematical Appendix

#### 6.1.1 Optimal Aggregate Consumption

In order to derive optimal aggregate consumption from individual Euler equations, iterate (5) forward to get:

$$\begin{aligned}\hat{c}_{t,j} &= \lim_{k \rightarrow \infty} E_t^j \hat{c}_{t+k,j} - \frac{1}{\sigma} E_t^j \sum_{k=0}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1}) \\ &= \hat{c}_{\infty}^j - \frac{1}{\sigma} E_t^j \sum_{k=0}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1})\end{aligned}\tag{29}$$

Aggregating across agents then gives:

$$\begin{aligned}
\hat{c}_t &= \lambda_t \hat{c}_t^{RE} + (1 - \lambda_t) \hat{c}_t^{SI} \\
&= \lambda_t \left( \hat{c}_\infty^{RE} - \frac{1}{\sigma} E_t^{RE} \sum_{k=0}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1}) \right) + (1 - \lambda_t) \left( \hat{c}_\infty^{SI} - \frac{1}{\sigma} E_t^{SI} \sum_{k=0}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1}) \right) \\
&= \lambda_t \hat{c}_\infty^{RE} + (1 - \lambda_t) \hat{c}_\infty^{SI} - \frac{1}{\sigma} \tilde{E}_t \sum_{k=0}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1}) \\
&= -\frac{1}{\sigma} \tilde{E}_t (\hat{i}_t - \pi_{t+1}) + \lambda_t \hat{c}_\infty^{RE} + (1 - \lambda_t) \hat{c}_\infty^{SI} - \frac{1}{\sigma} \tilde{E}_t \sum_{k=1}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1}) \\
&= \tilde{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \tilde{E}_t (\hat{i}_t - \pi_{t+1}) + \lambda_t \hat{c}_\infty^{RE} + (1 - \lambda_t) \hat{c}_\infty^{SI} - \frac{1}{\sigma} \tilde{E}_t \sum_{k=1}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1}) - \tilde{E}_t \hat{c}_{t+1} \\
&= \tilde{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \tilde{E}_t (\hat{i}_t - \pi_{t+1}) + \lambda_t \hat{c}_\infty^{RE} + (1 - \lambda_t) \hat{c}_\infty^{SI} - \frac{1}{\sigma} \tilde{E}_t \sum_{k=1}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1}) \\
&\quad - \tilde{E}_t (\lambda_t \hat{c}_\infty^{RE} + (1 - \lambda_t) \hat{c}_\infty^{SI} - \frac{1}{\sigma} \tilde{E}_{t+1} \sum_{k=1}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1})) \\
&= \tilde{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \tilde{E}_t (\hat{i}_t - \pi_{t+1}) + [\lambda_t \hat{c}_\infty^{RE} + (1 - \lambda_t) \hat{c}_\infty^{SI}] - [\tilde{E}_t (\lambda_t \hat{c}_\infty^{RE} + (1 - \lambda_t) \hat{c}_\infty^{SI})] \\
&\quad + [(-\frac{1}{\sigma}) \tilde{E}_t \sum_{k=1}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1})] - [(-\frac{1}{\sigma}) \tilde{E}_t \tilde{E}_{t+1} \sum_{k=1}^{\infty} (\hat{i}_{t+k} - \pi_{t+k+1})] \\
\hat{c}_t &= \tilde{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \tilde{E}_t (\hat{i}_t - \pi_{t+1}), \tag{30}
\end{aligned}$$

where  $\tilde{E}_t = \lambda_t E_t^{RE} + (1 - \lambda_t) E_t^{SI} = \lambda_t E_t + (1 - \lambda_t) \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j}$ .

### 6.1.2 Sticky Information Phillips Curve

Following [Ball et al. \(2005\)](#), the sticky information Phillips curve is derived as follows: Under flexible prices and full information, firms set relative prices equal to real marginal costs and the mark-up  $\mu \equiv \frac{\theta}{\theta-1}$ :

$$\left( \frac{p_{t,i}^*}{P_t} \right) = \mu \varphi_t = \frac{\theta}{\theta-1} \varphi_t \tag{31}$$

Also, the definition of real marginal costs and households' optimal deci-

sion regarding the allocation of leisure and the real wage gives:

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu} = \frac{N_t^\eta}{C_t^{-\sigma}}, \quad (32)$$

which gives an alternative expression of real marginal costs as  $\varphi_t = \frac{C_t^\sigma N_{t,i}^\eta}{Z_t}$ . Substituting this expression into (31) and assuming market clearing as well as the production function  $y_{t,i} = Z_t N_{t,i}$ , we get:

$$\left( \frac{P_{t,i}^*}{P_t} \right) = \frac{\theta}{\theta - 1} \frac{Y_t^\sigma \left( \frac{y_{t,i}}{Z_t} \right)^\eta}{Z_t} \quad (33)$$

Finally, substituting for  $y_{t,i}$  with the demand equation in (13) and taking logarithms gives:

$$\hat{p}_{t,i}^* = \hat{p}_t + \frac{\sigma + \eta}{1 + \eta\theta} \hat{y}_t - \frac{1 + \eta}{1 + \eta\theta} \hat{z}_t + e_t, \quad (34)$$

where again variables with a hat denote deviations from steady state and  $e_t$  is an i.i.d. shock that can be interpreted as a cost-push shock resulting, for instance, from wage or tax changes. Now, assuming fully competitive markets with complete information, where all firms set  $\hat{p}_{t,i}^* = \hat{p}_t$ , we get for the natural output under flexible prices  $y_t^n$ :

$$\hat{y}_t^n = \frac{1 + \eta}{\sigma + \eta} \hat{z}_t \quad (35)$$

Solving (35) for  $\hat{z}_t$  and substituting into (34) then gives the deviation of firm  $i$ 's optimal price from its steady state in terms of the aggregate price level and the output gap:

$$\hat{p}_{t,i}^* = \hat{p}_t + \frac{\sigma + \eta}{1 + \eta\theta} (\hat{y}_t - \hat{y}_t^n) + e_t \quad (36)$$

Accounting for the role of expectations under limited information derived in (14), we then get for the aggregate price index:

$$\begin{aligned}
\hat{p}_t &= \lambda_t E_t^{RE} [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t] + (1 - \lambda_t) E_t^{SI} [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t] \\
&= \tilde{E}_t [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t]
\end{aligned} \tag{37}$$

where  $\psi = \frac{\sigma + \eta}{1 + \eta\theta}$ . Finally, we lag equation (37) by one period and subtract it from (37), setting  $\lambda_t = \bar{\lambda}$ . After some algebra, we arrive at the sticky-information Phillips curve as in [Ball et al. \(2005\)](#):

$$\begin{aligned}
\hat{p}_t - \hat{p}_{t-1} &= \tilde{E}_t [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t] - \tilde{E}_{t-1} [\hat{p}_{t-1} + \psi (\hat{y}_{t-1} - \hat{y}_{t-1}^n) + e_{t-1}] \\
\pi_t &= \bar{\lambda} (\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t) + \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^{j+1} E_{t-1-j} [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t] \\
&\quad - \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} [\hat{p}_{t-1} + \psi (\hat{y}_{t-1} - \hat{y}_{t-1}^n) + e_{t-1}] \\
\pi_t &= \bar{\lambda} (\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t) - \bar{\lambda} \hat{p}_t - \frac{\bar{\lambda}^2}{1 - \bar{\lambda}} (\psi (\hat{y}_t - \hat{y}_t^n) + e_t) \\
&\quad + \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} [\pi_t + \psi \Delta (\hat{y}_t - \hat{y}_t^n) + \Delta e_t] \\
\pi_t &= \frac{\psi \bar{\lambda}}{1 - \bar{\lambda}} (\hat{y}_t - \hat{y}_t^n) + \frac{\bar{\lambda}}{1 - \bar{\lambda}} e_t \\
&\quad + \bar{\lambda} \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} [\pi_t + \psi \Delta (\hat{y}_t - \hat{y}_t^n) + \Delta e_t]
\end{aligned} \tag{38}$$

### 6.1.3 The Switching Mechanism

Recursive solutions to the infinite sums of past squared forecast errors are derived as follows:

From equation (19), we get when inserting the definition of  $\omega_k$ :

$$\begin{aligned}
U_t^{RE} &= - \sum_{k=0}^{\infty} [\omega_k (\hat{x}_{t-k} - E_{t-k-1} \hat{x}_{t-k})^2 + K^{RE}] \\
&= - \sum_{k=0}^{\infty} [(1-\rho)\rho^k (\hat{x}_{t-k} - E_{t-k-1} \hat{x}_{t-k})^2 + K^{RE}] \\
&= -(1-\rho) \sum_{k=0}^{\infty} [\rho^k (\hat{x}_{t-k} - E_{t-k-1} \hat{x}_{t-k})^2 + K^{RE}] \quad (39)
\end{aligned}$$

Lagging equation (39) by one period gives:

$$U_{t-1}^{RE} = -(1-\rho) \sum_{k=0}^{\infty} [\rho^k (\hat{x}_{t-k-1} - E_{t-k-2} \hat{x}_{t-k-1})^2 + K^{RE}] \quad (40)$$

Now, we get when extracting the first term of the infinite sum in (39) and using (40):

$$\begin{aligned}
U_t^{RE} &= -(1-\rho) \sum_{k=0}^{\infty} [\rho^{k+1} (\hat{x}_{t-k-1} - E_{t-k-2} \hat{x}_{t-k-1})^2 + K^{RE}] \\
&\quad - (1-\rho) [(\hat{x}_t - E_{t-1} \hat{x}_t)^2 + K^{RE}] \\
U_t^{RE} &= \rho U_{t-1}^{RE} - (1-\rho) [(\hat{x}_t - E_{t-1} \hat{x}_t)^2 + K^{RE}] \quad (41)
\end{aligned}$$

Derivations for  $U_t^{SI}$  apply in the same manner.

## 6.2 Figures

Figure 1: Impulse Responses to a Demand Shock

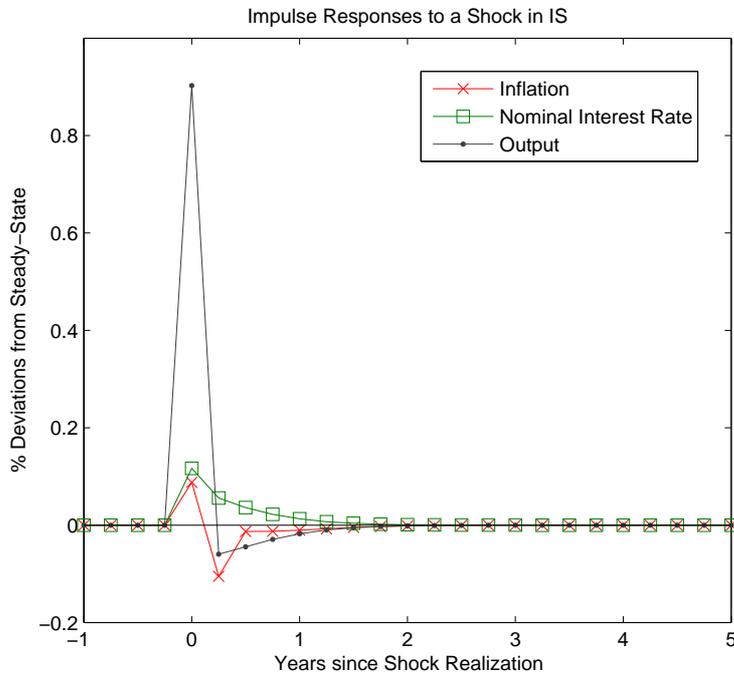


Figure 2: Impulse Responses to a Cost-Push Shock

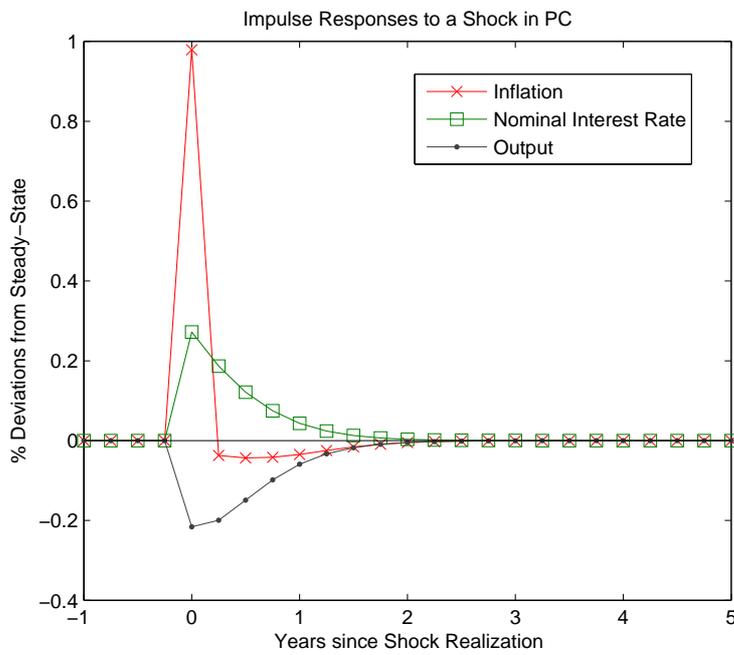


Figure 3: Impulse Responses to a Monetary Policy Shock

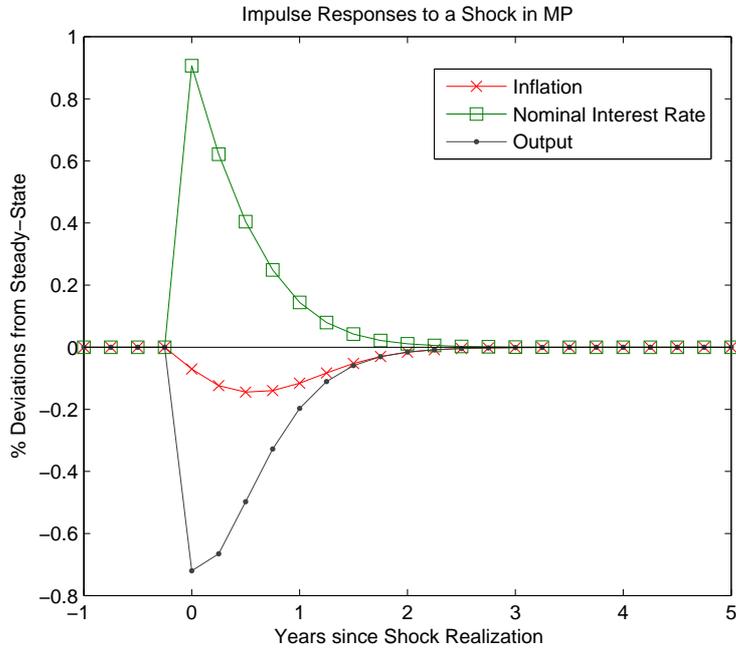


Figure 4: Impulse Responses to a Technology Shock

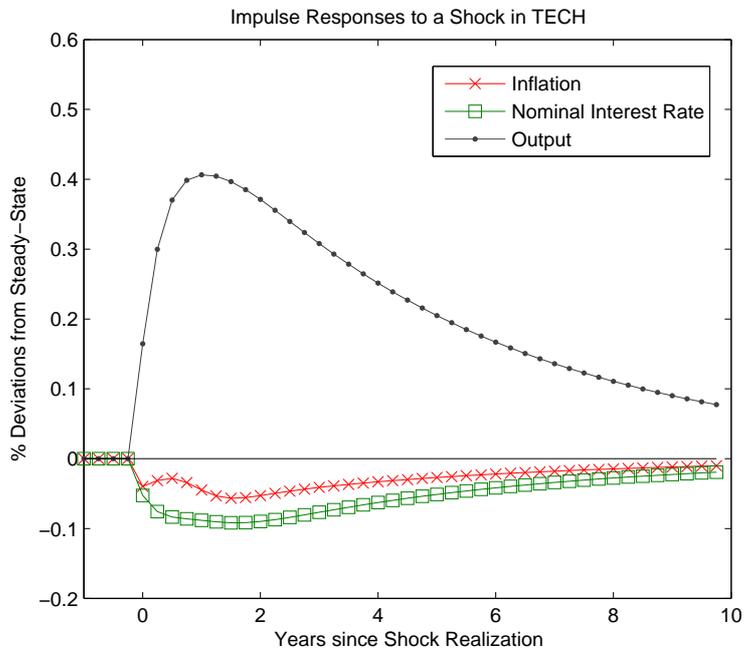


Figure 5: Optimal Policy across Taylor-Rule Coefficients

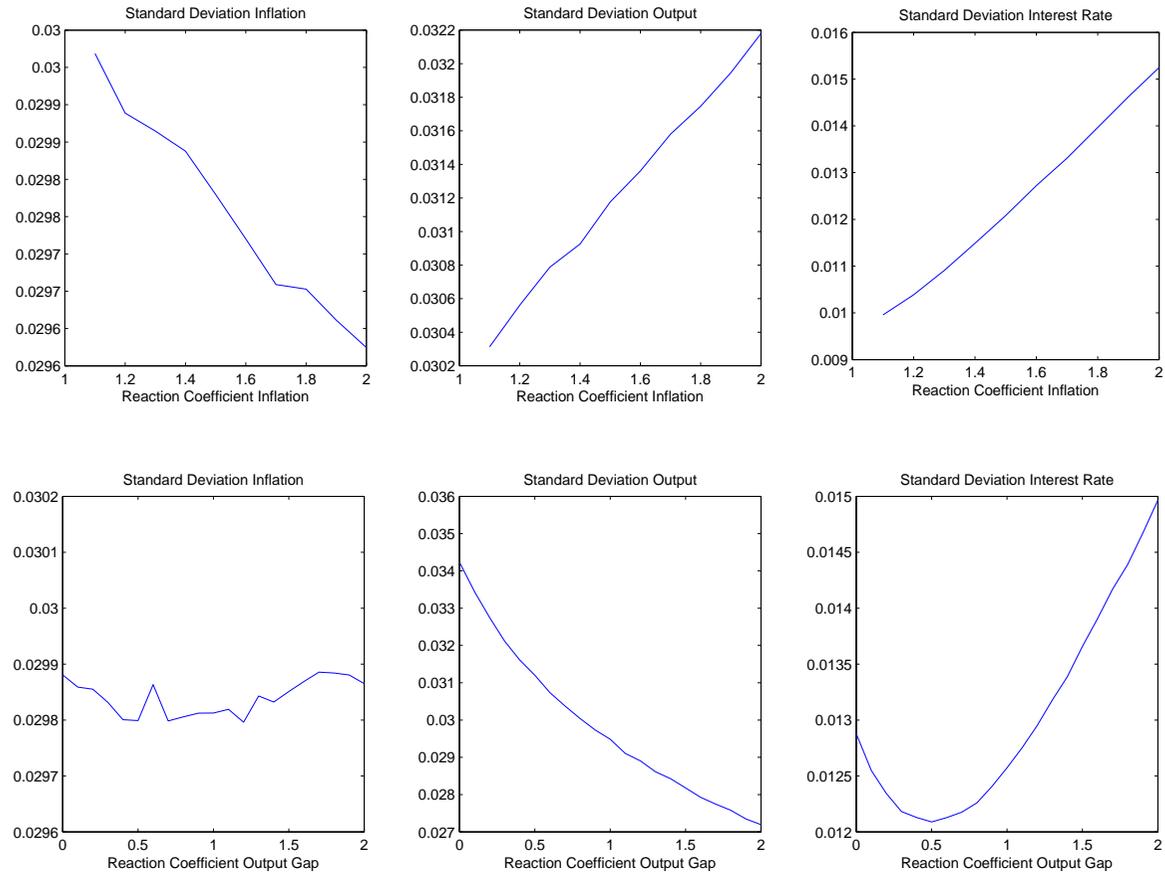


Figure 6: Impulse-Responses with Varying Reaction Coefficients to Inflation

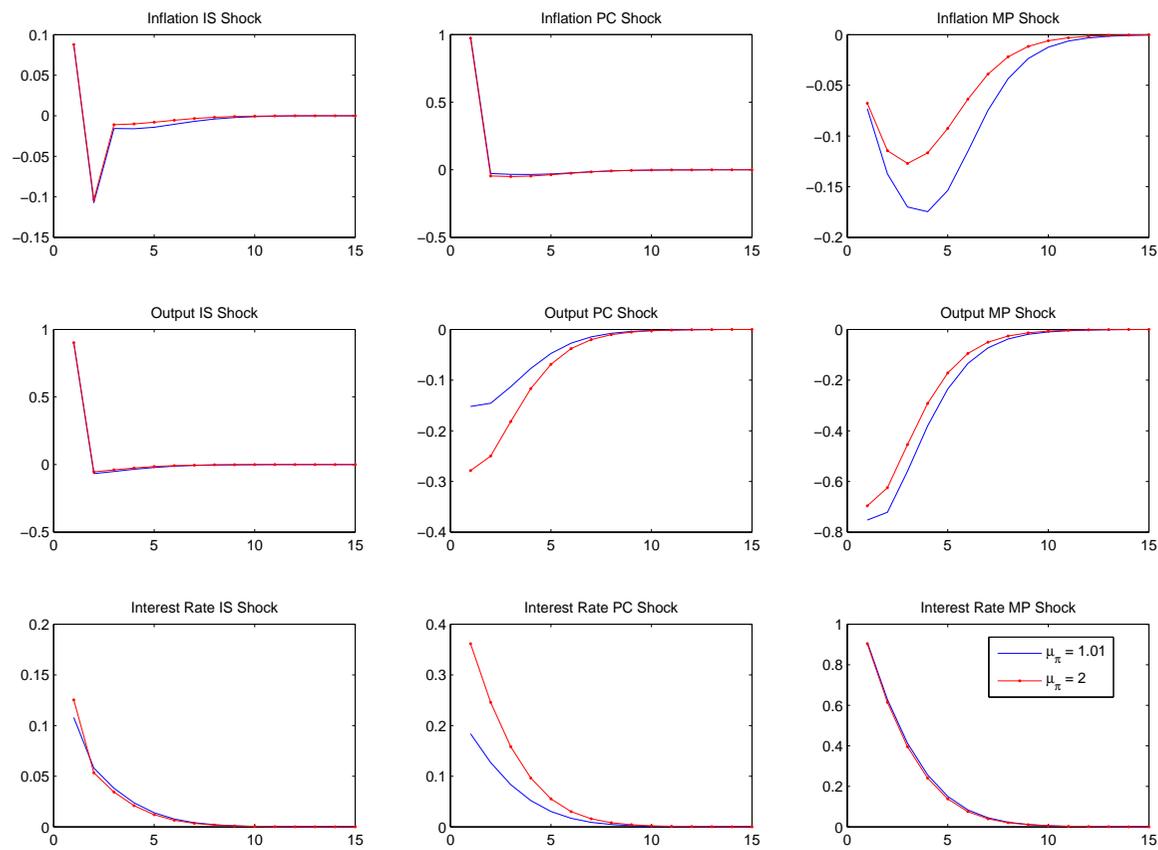
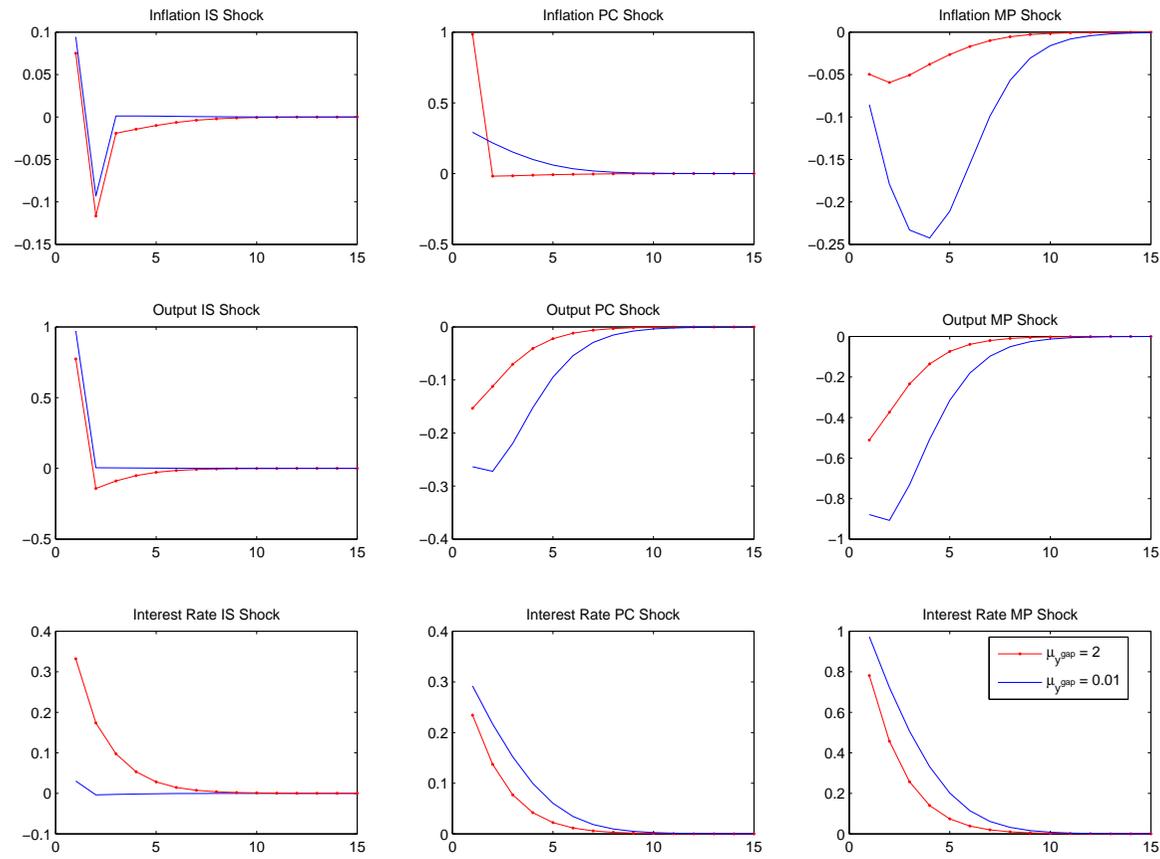


Figure 7: Impulse-Responses with Varying Reaction Coefficients to the Output Gap



### 6.3 Tables

Table 1: Statistics of Simulated Variables

Variable	Standard Deviation	AR(1) coefficient
$\pi_t$ K=0%	0.0298	-0.0491
K=10%	0.0298	-0.0201
K=25%	0.0298	-0.0186
K=50%	0.0298	-0.0496
K=75%	0.0298	-0.0615
K=100%	0.0298	-0.0506
$\hat{y}_t$ K=0%	0.0312	0.1337
K=10%	0.0311	0.1288
K=25%	0.0312	0.1554
K=50%	0.0312	0.1525
K=75%	0.0312	0.1588
K=100%	0.0312	0.1761
$\hat{i}_t$ K=0%	0.0121	0.6232
K=10%	0.0121	0.6401
K=25%	0.0121	0.6312
K=50%	0.0121	0.6482
K=75%	0.0121	0.6733
K=100%	0.0121	0.6526

Note: Values are obtained from simulating the model 1000 times over 1000 periods.

Table 2: Distribution of the Shares of Rational Agents

Variables	Min.	Max.	Mean	Std.	Av. Cycle Length
$\lambda_t^y$ K=0%	0.6667	1	0.9935	0.0414	9.403
K=10%	0.2152	1	0.9578	0.1216	7.241
K=25%	0.0052	1	0.8085	0.3232	5.782
K=50%	0.0000	1	0.4842	0.4651	2.974
K=75%	0.0000	1	0.2506	0.4223	4.690
K=100%	0.0000	1	0.1229	0.3119	10.789
$\lambda_t^\pi$ K=0%	0.5084	1	0.9931	0.0390	8.709
K=10%	0.1008	1	0.9421	0.1414	8.438
K=25%	0.0011	1	0.7283	0.4159	5.260
K=50%	0.0000	1	0.3439	0.4607	4.132
K=75%	0.0000	1	0.1339	0.3250	8.046
K=100%	0.0000	1	0.0486	0.2038	21.390

Note: The mean is obtained from simulating the model 1000 times over 1000 periods. The average switching frequency is calculated in quarters.

Table 3: Correlation of  $\lambda_t$  with Macroeconomic Variables

Correlation with	$\lambda_t^y$	$\lambda_t^\pi$
level $\pi_t$	-0.0302	-0.0499
variance $\pi_t$	0.0369	0.3958
level $y_t$	0.0805	0.0245
variance $y_t$	0.3339	-0.1015
level $i_t$	-0.1012	-0.0332
variance $i_t$	0.0477	0.2512

Simulated with  $K^{RE} = 50\%$ .

Table A1: Parameter Values for Calibration

Parameters		Baseline Model: McCallum (2001)
intertemporal elasticity of substitution	$1/\sigma$	0.4
coefficient of relative risk aversion	$\sigma$	2.5
Inverse of Frisch elasticity of labour supply	$\eta$	1
elasticity of substitution between goods	$\theta$	35
coefficient on output gap from PC	$\psi = \frac{\sigma+\eta}{1+\eta\theta}$	0.097
weight of inflation target Taylor rule	$\mu_\pi$	1.5
weight of output Taylor rule	$\mu_{y^{gap}}$	0.5
interest smoothing Taylor rule	$\mu_i$	0.8
AR term technology shock	$\alpha_4$	0.950
std IS shock	$\tau_1$	0.030
std PC shock	$\tau_2$	0.030
std MP shock	$\tau_3$	0.0017
std technology shock	$\tau_4$	0.007
<i>Calibration of switching parameters:</i>		
initial share of rational output expectations	$\lambda^y$	0.5
initial share of rational inflation expectations	$\lambda^\pi$	0.5
intensity of choice (Brock/Hommes 1997)	$\gamma$	10000
memory of past forecast errors (De Grauwe 2008)	$\rho$	0.5

Table A2: Costs of Rationality

percentage of	mean $U_{y,t}^{SI}$	mean $U_{\pi,t}^{SI}$
100%	0.002500	0.003000
75%	0.001875	0.002250
50%	0.001250	0.001500
25%	0.000625	0.000750
10%	0.000250	0.000300
0%	0.000000	0.000000

Note: Mean values are generated from a simulation of the baseline model over 1000 periods with  $\rho = 0$  and  $K^{RE} = 0$ .

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