Measuring Macroeconomic Convergence and Divergence within EMU Using Long Memory

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Abstract

This paper measures the convergence or divergence of EMU inflation rates and industrial production by testing for the existence of fractional cointegration relations. The notion of fractional cointegration allows for long-term equilibria with a higher degree of persistence than allowed for in the standard cointegration framework. We investigate both inflation and industrial production of EMU countries beginning with the introduction of the common currency and including the financial crisis and post-crisis period. Core as well as periphery countries are included in the study. By modelling possible breaks in the persistence structure we find evidence of fractional cointegration as well as a lower persistence before the crisis and a higher persistence by less evidence for fractional cointegration during the crisis. A second break which indicates the end of the crisis can be found as well. In addition, higher inflation persistence can be found for periphery than for core countries.

Keywords: EMU inflation rates, industrial production, fractional cointegration, persistence breaks

JEL classification: F15, F45, C32

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1 Introduction

Ever since the idea of a European Monetary Union (EMU) was born, economists have discussed the need for member countries to converge in nominal and real terms in order ensure the stability of the union. This follows from the concern that inflation differentials between member countries with a single monetary policy may lead to differences in real interest rates and in international competitiveness, thereby creating real divergence as measured in current account imbalances. These imbalances are difficult to address if the affected countries share a common currency and thus have permanently fixed nominal exchange rates and a single monetary authority (Mundell, 1961).

Early studies before the start of EMU demonstrate some success in terms of nominal convergence of the member states (Beliu and Higgins, 2004), whereas others use cointegration analysis to demonstrate potential long-run stability problems with respect to macroeconomic dynamics in the so-called "periphery" countries Italy, Spain and Portugal (Haug et al., 2000). However, the initial convergence in inflation rates was somewhat reversed after the start of EMU, resulting in persistent inflation differentials, where groups of countries showed inflation rates persistently above or below the EMU average (ECB, 2003). While some authors argue that these dynamics were signs of economic catching-up processes (Blanchard and Giavazzi, 2002), the European sovereign debt crisis following the financial crisis of 2008 revealed potential large economic costs of macroeconomic divergence within a monetary union (Borio and Disyatat, 2011; Gnimassoun and Mignon, 2016).

In this paper, we thus use fractional cointegration methods accounting for long-memory equilibrium processes as a tool to measure nominal and real convergence or divergence since the start of EMU. Testing for the existence of a long-run equilibrium between nominal and real macroeconomic variables across EMU countries with fractional cointegration has the advantage of allowing the degree of integration to take any real number on the unit interval. Therefore, the framework allows for higher flexibility by allowing the long-run equilibrium to be mean-reverting rather than a short-memory stationary process, which has proven to be a too restrictive assumption. Evidence for fractional cointegration with long memory in the equilibrium errors then indicates the existence of a long-run equilibrium, while at the same time deviations from this equilibrium may be very persistent. In contrast to the previous literature, we account for the full EMU period including the financial crisis period from 2008 onwards. Including the financial crisis period though makes our analysis vulnerable to spurious results through changes in the convergence mechanism and thus in the cointegrating relation. We therefore apply the regression-based Lagrange Multiplier test introduced by Martins and Rodrigues (2014) to test for structural breaks in the order of integration of the inflation rates and industrial production and estimate the break dates. As this may also affect the cointegrating relation, we also consider the pre- and post-crisis periods separately.

We estimate the degree of fractional long memory for inflation and industrial production in 11 EMU countries for the period from January, 1999, to June, 2019, where we distinguish between "core" and "periphery" countries. Our results suggest breaks in the persistence of both inflation and industrial production around the beginning of the financial crisis in 2007-08 for a majority of countries in our sample. Inflation is generally found to be more persistent in the crisis period, with higher persistence estimated for periphery countries compared to core countries. This implies that inflation rates where more integrated in the EMU during the pre-crisis period, while the higher persistence during the crisis period carries the danger of diverging processes in case of asymmetric inflation shocks. By contrast, industrial production is found to be more persistent in the core countries of the sample. Again, most countries experienced higher persistence also in industrial production during the crisis, but Belgium, Finland and Greece show the opposite pattern with lower persistence after 2008.

Our paper builds on previous work by some of the authors in Leschinski et al. (2018). Using the fractional cointegration methodology, Leschinski et al. (2018) provide empirical evidence for periods of convergence and divergence for long-term EMU government bonds that coincide with bull- and bear-market periods in the stock market. Specifically, stronger market integration is associated with bull-market periods and is more intense among core countries than among periphery countries. Periods of disintegration coincide with bear-market periods. Their results thus imply time-variation in the degree of convergence of EMU government bonds, with the possibility of divergence even before the financial crisis of 2008. Moreover, the authors report evidence of disintegration in government bonds for all countries during the period of the financial crisis and the European sovereign debt crisis from 2008 onwards.

We thus also test for the existence of fractional cointegration relationships for inflation and IP among all country pairs of our sample and estimate the reduction rate b of the fractional cointegration relation using the Souza et al. (2018) test for the full sample. We find evidence of fractional cointegration for both inflation and industrial production, with a larger number of cointegration relationships in the case of inflation country-pairs. By contrast, the strength of cointegration found for industrial production is stronger than those for inflation. Overall, it seems that inflation rates are cointegrated primarily among core or among periphery countries, but we find evidence of mixed core-periphery convergence clubs for industrial production.

Our paper further relates to the previous literature using fractional cointegration methods to measure macroeconomic convergence between European economies. In an early study, Beliu and Higgins (2004) use fractional cointegration tests to evaluate the convergence of inflation, long-term interest rates and industrial production of 14 EU countries vis-à-vis Germany. The sample (1957-2001) covers the period until the Euro cash changeover, whereas we focus on the period after the start of EMU in 1999. The authors present evidence of nominal convergence in inflation and long-term interest rates, as these

series are fractionally cointegrated with the German counterpart. However, the equilibrium errors display long memory, so that any deviation from equilibrium will be persistent. The authors find no evidence of fractional cointegration in industrial production and, thus, no evidence for real convergence. Meller and Nautz (2012) test for differences in inflation dynamics among European countries before and after EMU using panel estimates of fractional cointegration. Their results suggest that inflation persistence converged and was significantly reduced with the introduction of EMU. More recently, Hualde and Iacone (2017b) analyze both level inflation as well as inflation differentials between EMU country pairs allowing for long memory and cointegration with the test procedure derived in Hualde and Iacone (2017a). Their results suggest that the "core" economies of EMU are more integrated than the "periphery" countries, as the latter show more persistent inflation differentials. Similarly, Karanasos et al. (2016) study convergence of inflation among EMU countries for the period 1980-2013 using a broad range of test methods, which includes tests allowing for long memory and for structural breaks. Similar to the study by Hualde and Iacone (2017b), the authors present evidence for three clubs of convergence consisting of "core" EMU countries, while there is evidence of divergence in inflation for the remaining countries. We extend these previous studies, as we account for the effects on nominal and real convergence of the recent period of economic turmoil following the financial crisis in 2008. In addition, we compare the results from several semiparametric tests for fractional cointegration by Souza et al. (2018), Wang et al. (2015) and Chen and Hurvich (2006).

Other studies applying the concept of fractional cointegration evaluate, for instance, the stability of money demand functions (Caporale and Gil-Alana, 2005), the effect of inflation targeting regimes on inflation persistence (Canarella and Miller, 2017) or the effect of a monetary policy shock when long memory in the output gap and inflation is accounted for (Lovcha and Perez-Laborda, 2018).

The remainder of the paper is structured as follows. The econometric methodology of fractional cointegration with long memory is detailed in section 2. Section 3 describes the data and some initial results from an earlier study on fractional cointegration of long-run interest rates within EMU. The results of our anaylsis are presented and discussed in section 4. Finally, section 5 summarizes and concludes.

2 Methodology

Inflation persistence in Europe has attracted renewed attention since the foundation of EMU and the implementation of a common monetary policy among the member countries. Persistence is here measured by the order of integration of the inflation series. If this order of integration d fulfills $0 < d < 0.5$ the series is said to exhibit stationary long memory or stationary long-range dependence resulting in a hyperbolic decay of the autocorrelation function and therefore in a slow decay of dependencies between observations far away from

each other. Also the impulse-responses show a hyperbolic decay. For $0.5 < d < 1$ the series is non-stationary but still mean reverting in the sense that the expected time for returning to its mean is finite. The concept of long memory or fractional integration widens therefore the traditional $I(1)/I(0)$ duality by allowing for highly persistent equilibrium processes.

A significant number of empirical studies that have been adopted in recent years support the existence of long-range dependence in inflation rates. Baillie et al. (2002), Canarella and Miller (2016), Canarella and Miller (2017) Hassler and Wolters (1995), Kumar and Okimoto (2007) and Meller and Nautz (2012) among others, measure persistence as a fractionally integrated process. Moreover, fractional integration techniques are able to capture convergence among inflation policies and a more consistent detection of persistence shifts than other econometric approaches. A weak definition of inflation convergence requires the existence of a stable equilibrium relationship, but not exact equality between the inflation rates. A similar definition of convergence can also be applied to real economic variables, such as industrial production (IP).

In general, (fractional) cointegration is an equilibrium concept where the persistence of the cointegrating residual d_v determines the speed of adjustment towards the cointegration equilibrium $\beta' X_t$, and shocks have no permanent influence on the equilibrium as long as $d_v < 1$ holds. We therefore allow for fractional cointegration and consider a bivariate system of the form

$$X_{1t} = c_1 + \xi_1 Y_t + \Delta^{-(d-b_1)} u_{1t} \mathbb{1}_{\{t>0\}} \tag{1}$$

$$X_{2t} = c_2 + \xi_2 Y_t + \Delta^{-(d-b_2)} u_{2t} \mathbb{1}_{\{t>0\}} \tag{2}$$

$$Y_t = \Delta^{-d} e_t \mathbb{1}_{\{t>0\}}, \tag{3}$$

where the coefficients c_1, c_2, ξ_1, and ξ_2 are finite, $0 \leq b_1, b_2 \leq d$, L is the lag-operator, the fractional differences $\Delta^d Y_t = (1-L)^d Y_t$ are defined in terms of generalized binomial coefficients such that

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-1)^k L^k = \sum_{k=0}^{\infty} \pi_k L^k,$$

$$\text{with} \quad \binom{d}{k} = \frac{d(d-1)(d-2)\dots(d-(k-1))}{k!},$$

and e_t and $u_t = (u_{1t}, u_{2t})'$ are martingale difference sequences. The memory of both X_{1t} and X_{2t} is determined by Y_t so that they are integrated of the same order d, denoted by $X_t \sim I(d)$, where the memory parameter is restricted to $d \in (0, 1]$ and $X_t = (X_{1t}, X_{2t})'$. Since it is assumed that $u_{1t} = u_{2t} = e_t = 0$ for all $t \leq 0$, the processes under consideration are fractionally integrated of type-II. For a detailed discussion of type-I and type-II

processes refer to Marinucci and Robinson (1999). The spectral density of X_t can be approximated by

$$f_X(\lambda) \sim \Lambda_j(d)\, G\, \overline{\Lambda_j(d)}, \quad \text{as } \lambda \to 0^+, \tag{4}$$

where G is a real, symmetric, finite, and positive definite matrix, $\Lambda_j(d) = \text{diag}\left(\lambda^{-d}e^{i\pi d/2}, \lambda^{-d}e^{i\pi d/2}\right)$ is a 2×2 diagonal matrix and $\overline{\Lambda_j(d)}$ is its complex conjugate transpose. The periodogram of a process X_t is defined through the discrete Fourier transform $w_X(\lambda_j) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} X_t e^{i\lambda_j t}$ as $I_X(\lambda_j) = w_X(\lambda_j)\overline{w_X(\lambda_j)}$, with Fourier frequencies $\lambda_j = 2\pi j/T$ for $j = 1, ..., \lfloor T/2 \rfloor$, where the operator $\lfloor \cdot \rfloor$ returns the integer part of its argument.

The two series X_{1t} and X_{2t} are said to be fractionally cointegrated, if there exists a linear combination

$$\beta' X_t = v_t,$$

so that the cointegrating residuals v_t are fractionally integrated of order $I(d-b)$ for some $0 < b \leq d$. Obviously, for the model in equations (1) to (3), this is the case for every multiple of the vector $\left(1, -\frac{\xi_1}{\xi_2}\right)'$ and $b = \min(b_1, b_2)$.

We restrict ourselves to a bivariate set-up as is common in the literature to avoid identification problems.

Here, we conclude that EMU inflation rates or EMU log industrial production levels, which are fractionally cointegrated with each other, are considered as economically integrated. From the definition above, this is the case if there exists an equilibrium relationship between the inflation or between industrial production (X_{1t} and X_{2t}), such that the persistence of deviations from the equilibrium denoted by v_t is reduced compared to that of the individual series. The degree of long memory $d-b$ in the cointegrating residual then determines the persistence of deviations from the long-run equilibrium. This implies that series, which show a high degree of long memory in the cointegrating residual are less firmly integrated than series with a low $d - b$.

3 Data

We conduct the analysis of macroeconomic convergence for 11 EMU countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain. As is common in the literature, we term the group of Austria, Belgium, Finland, France, Germany and the Netherlands the 'core countries' of EMU, while we call the group of Greece, Italy, Ireland, Spain and Portugal the 'periphery countries'. The monthly sample ranges from 1999:1 to 2019:6, with 2015 as base year (2015 = 100). Our sample thus starts with the official start of the EMU with the ECB acting as single central bank for the monetary union.

Data for inflation is obtained from Eurostat and measured with the monthly seasonally unadjusted Harmonized Indices of Consumer Prices for all items. We seasonally adjust each series by using the X-13 R package, developed by the United States Census Bureau. Then, we define annualized inflation rates for each country i as

$$\pi_{it} = 1200(log(HCPI_{it}) - log(HCPI_{it-1}))$$

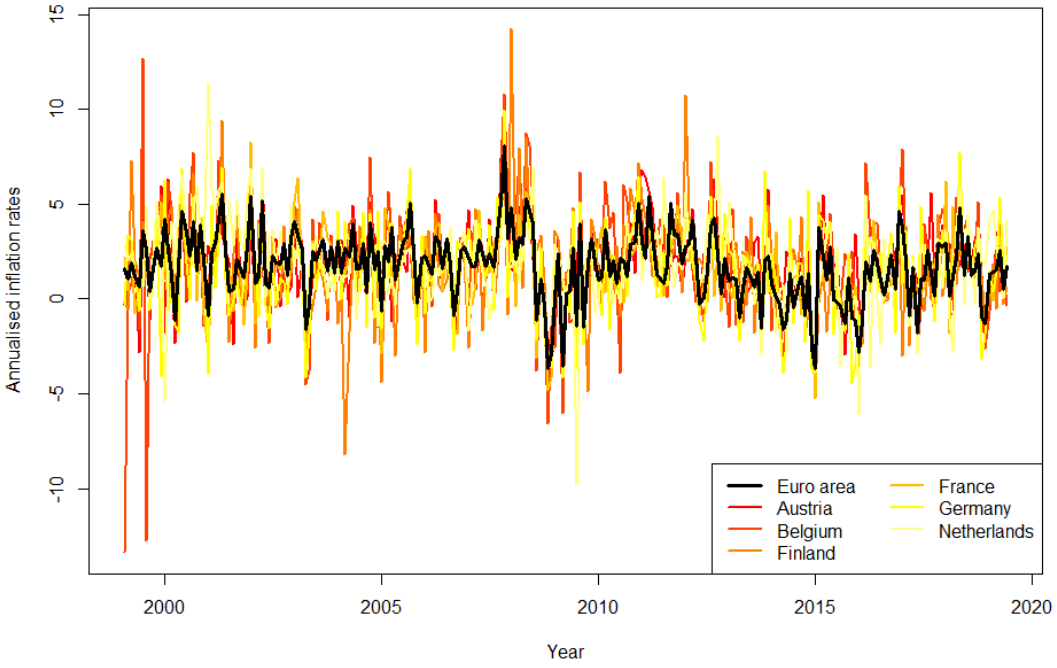
The series are shown for all countries in our sample in Figure 1. Visual inspection suggests that all EMU countries in our sample experienced a drop in inflation at the start of the financial crisis in 2008, with somewhat more volatile inflation rates after 2008. Overall, inflation rates in the periphery countries seem more volatile than those in the core countries.

Data for real monthly seasonally adjusted total industrial production indices are obtained from the Federal Reserve Economic Database (FRED) by the St. Louis Fed with 2015 = 100 as a base year. We employ the log of the industrial production for each country. Last, we detrend each series by applying a kernel smoothing regression. The data is shown in Figure 2. As in the case of inflation rates, the pronounced drop in industrial production in 2008 is clearly visible in all countries of our sample, except Ireland. Before the crisis, countries like France, Germany and Spain experienced very stable IP, while countries like Finland, Greece and Portugal exhibit higher IP volatility. After the financial crisis, the negative effects of the European sovereign debt crisis on real industrial production are clearly observable in Greece, Ireland and Portugal.

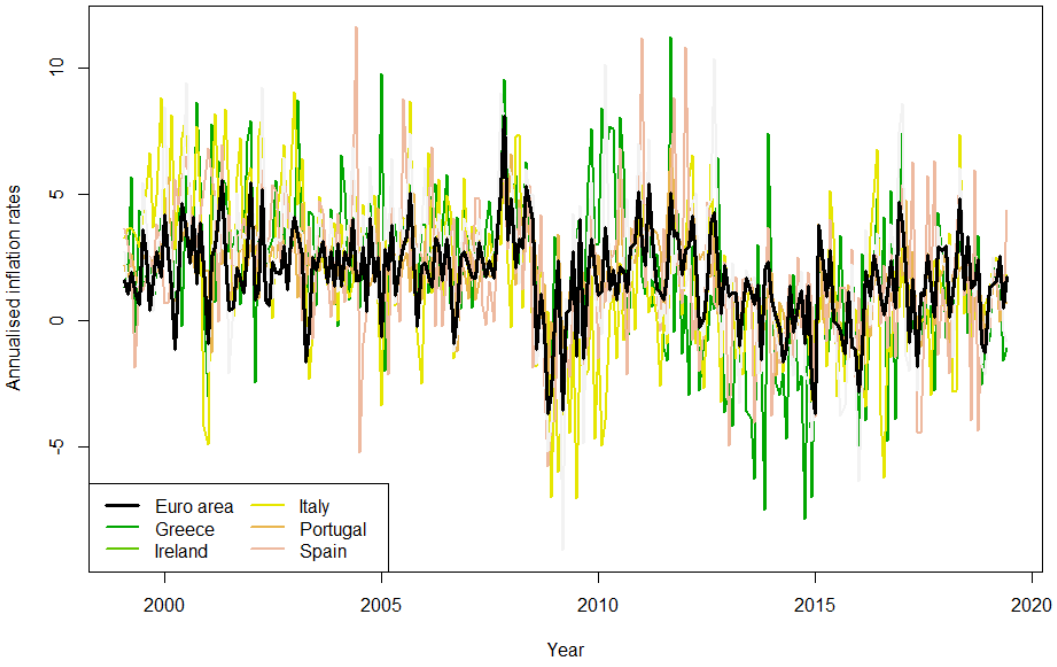
In order to account for the effects of the global financial crisis and the European sovereign debt crisis on macroeconomic convergence with EMU, we divide our analysis in two sub-periods. The financial crisis started with the subprime crisis in the US in 2007, but became a global financial crisis with the default of Lehman Brothers on September 15, 2008. Therefore, we compare the dynamics of inflation before and after this seminal event. The pre-crisis period refers to the beginning of the EMU until August of 2008. Next, we evaluate the crisis and post-crisis period from September 2008-June 2019. Note that the crisis period includes both the global financial crisis, as well as the European sovereign debt crisis, which severely affected several European countries during 2010-2012. Finally, we also evaluate the full sample period. Table 1 summarizes the exact dates of our analysis.

Table 1: Definition of sub-periods.

Periods	Start	End
Pre-Crisis	Jan 1999	August 2008
Crisis/Post-Crisis	September 2009	June 2019
Full Sample	Jan 1999	June 2019

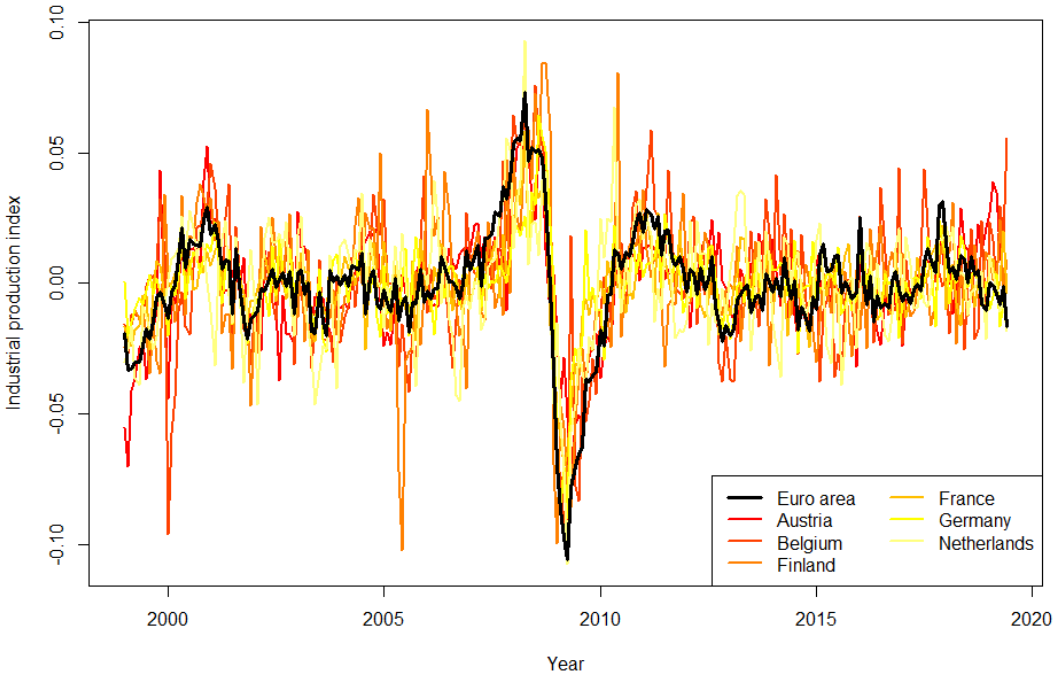


(a) Inflation rates in core countries

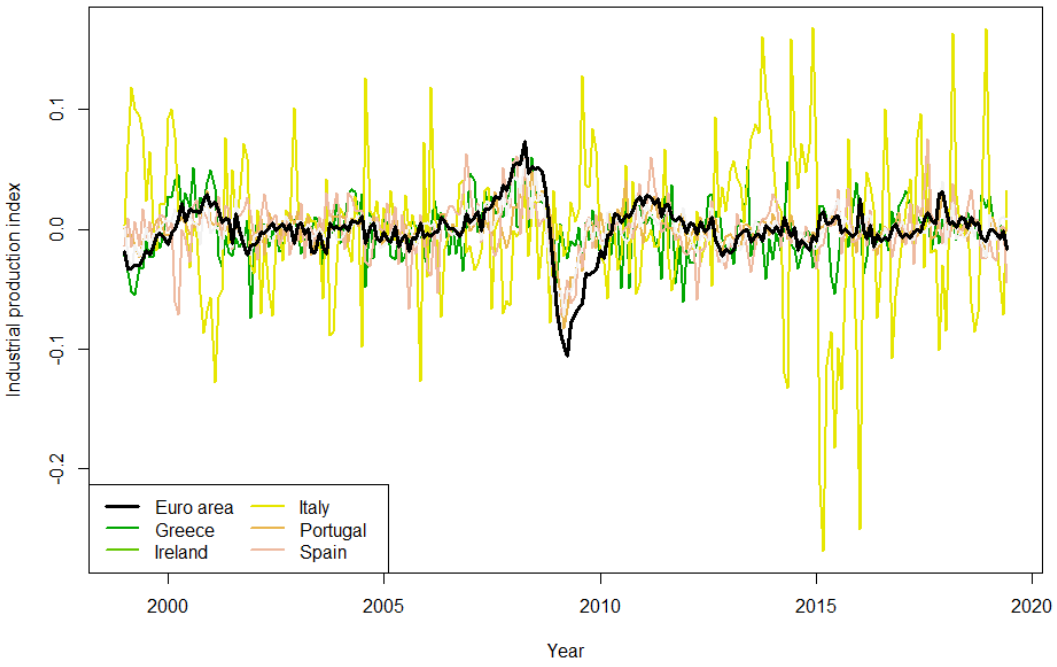


(b) Inflation rates in periphery countries

Figure 1: Inflation rates of EMU countries.



(a) Industrial production in core countries



(b) Industrial production in periphery countries

Figure 2: Detrended log industrial production of EMU countries.

4 Empirical Results

4.1 Analysis of Convergence and Divergence of EMU Inflation Rates

4.1.1 Measuring the degree of long memory in EMU inflation rates

In this section, following Kumar and Okimoto (2007), we model inflation rates as a fractionally integrated process and estimate the memory parameters as a measure of persistence for the largest economies in the EMU. Memory parameters for each country and sub-period are shown in Table 2, where we differentiate between the so-called "core" economies of Austria, Belgium, Finland, France, Germany and the Netherlands, and the so-called "periphery" countries Greece, Ireland, Italy, Portugal and Spain. For our estimation, we use the exact local Whittle estimator of Shimotsu (2010) with a 0.75 bandwidth d-value ($m = T^{0.75}$). As a direct extension of Shimotsu et al. (2005), this estimator has the advantage of allowing for non-zero means, while the properties of consistency and asymptotically normal distribution for all values of d continue to hold. Similar to other findings in the literature (Hualde and Iacone, 2017a; Meller and Nautz, 2012), inflation persistence is higher for the periphery countries when we refer to the full sample period. In fact, the mean values of the order of fractional integration are 0.23 and 0.33 for core and periphery countries, respectively. This implies that the inflation rates among core countries are more firmly integrated than among the group of periphery countries.

However, the sub-samples under examination present an interesting variability. The pre-crisis shows negative or close to zero memory values for most of the countries, which confirms previous findings of reduced inflation persistence in the beginning of EMU (Meller and Nautz, 2012; Karanasos et al., 2016). A notable exception is the Netherlands during this period with higher levels of inflation persistence already before the financial crisis. In general, the low levels of long memory in inflation and the similar memory parameters across EMU countries suggest convergence in inflation rates in the early years of EMU.

Comparing the pre-crisis period with the crisis/post-crisis period from 2008-2019, we observe that inflation persistence is estimated to be substantially higher in the crisis and post-crisis period. This implies that possible divergence from any equilibrium relationship between inflation rates of EMU members would be much longer-lasting. We test this issue further in the next section, where we explicitly test for fractional cointegration in inflation rates. In addition, the results in Table 2 show a diversion in estimated inflation persistence between the core and periphery economies during the crisis/post-crisis period. On average, the periphery countries exhibit higher memory parameters, thus higher persistence, in inflation rate in the crisis period than the core countries. Overall, the range of memory parameters is larger in the crisis period, ranging from 0.23 in Germany to marginally non-stationary mean reverting values for Spain (d=0.5) and France (d=0.68).

Table 2: Memory estimates for inflation

EMU country	Pre-crisis	Crisis/Post-crisis	Full sample
Austria	0.03	0.30	0.23
Belgium	0.12	0.33	0.22
Finland	0.15	0.38	0.29
France	-0.05	0.68	0.21
Germany	0.01	0.23	0.11
Netherlands	0.38	0.43	0.32
Greece	-0.09	0.36	0.36
Ireland	0.13	0.32	0.35
Italy	0.23	0.46	0.43
Portugal	0.06	0.40	0.27
Spain	0.01	0.50	0.26
Mean	0.09	0.40	0.28

Note: Exact local Whittle estimates of d with bandwidth $m = T^{0.75}$.

Next, we test for a break in the order of fractional integration in inflation rates. Hence, we perform a regression-based Lagrange Multiplier test introduced by Martins and Rodrigues (2014) that generalizes the conventional integration approaches to the fractional integrated process context. In what follows we will denote by $\hat{\tau}$ the estimated point of the persistence shift and by d_1 the order of integration before the shift and by d_2 the order of integration after the break. Table 3 suggests evidence of a structural break in inflation persistence for almost all EMU countries. Specifically, only two countries (Netherlands and Portugal) do not reject the null hypothesis of constant persistence against the alternative of an increase in the memory parameter. Austria, Belgium, France, Greece, Ireland and Italy's estimated breakpoints ($\hat{\tau}$) occur from October, 2006 to October, 2008. Comparing d_1 and d_2, the results suggest a shift from $d \leq 0$ to $d > 0$ at the time of $\hat{\tau}$, that is an increase in inflation persistence after the break. The alternative hypothesis of decreasing memory holds for a smaller number of countries. For Finland, Greece, Ireland, Italy, Portugal and Netherlands we find (additional) memory shifts between April, 2014 and April, 2015, with lower levels of persistence after the break. Overall, the results in Table 3 reinforce our findings from Table 2 that inflation persistence increased during the crisis period after 2008. For some countries, we find additional evidence that the crisis period ended around 2014/2015, with lower inflation persistence returning to lower levels.

4.1.2 Testing for fractional cointegration in EMU inflation rates

We apply a number of semiparametric tests for the null hypothesis of no fractional cointegration. The advantage of semiparametric methods is that we do not impose any assumptions on the short-run behavior of the series, apart from mild regularity conditions. Thus, we can avoid spurious findings that might arise due to misspecification. Research

Table 3: Persistence change test for inflation

EMU country	Test-statistic	Date	d_1	d_2
Austria	-3.86***	10/2006	0.02	0.26
Belgium	-2.65***	09/2007	0.02	0.32
Finland	-2.18**	04/2003	-0.22	0.34
	-1.78*	04/2015	0.27	0.16
France	-1.84*	07/ 2007	-0.24	0.41
Germany	-2.93***	06/2005	-0.19	0.14
Netherlands	-1.76*	12/2014	0.34	0.17
Greece	-2.95***	05/2007	-0.04	0.36
	-3.76***	10/2014	0.42	-0.08
Ireland	-2.07**	10/2008	0.19	0.20
	-2.21**	02/2012	0.40	-0.15
Italy	-4.69***	08/2007	0.00	0.48
	-1.77*	12/2014	0.40	0.24
Portugal	-3.32***	04/2014	0.35	-0.03
Spain	-1.75*	06/2005	-0.22	0.27

Note: *, **, *** indicate levels of significance at 10%, 5%, 1%, respectively. "d_1" and "d_2" refer to the memory parameters of the respective subperiod of the estimated breakpoint.

on semiparametric tests for fractional cointegration has been an active field in recent years and there exist a variety of competing approaches. Whereas some approaches rely on the spectral representation of multivariate long memory processes and test whether the spectral matrix G has full rank or not, other tests are residual-based and test for the strength of integration of the cointegration residuals. To make sure that our results are robust to the way of testing, we apply tests from both strands of the literature.

Souza et al. (2018) use the fractionally differenced process $\Delta^d X_t$ and the fact that the determinant $D_{\Delta^d}(\lambda)$ of $f_{\Delta^d X}(\lambda)$ is of the form $D_{\Delta^d}(\lambda) \sim \tilde{G}|1 - e^{-i\lambda}|^{2b}$, where $\tilde{G}$ is a scalar constant and $0 < \tilde{G} < \infty$. An estimate of b can therefore be obtained via a log-periodogram regression and the null hypothesis that $b = 0$ can be tested based on the resulting estimate. We denote this spectral-based test in the following as SRFB18.

The test of Wang et al. (2015) (denoted by WWC15) is based on the sum over the fractionally differenced process $\Delta^{\hat{d}_v} X_{2t}$, where $\hat{d}_v$ is an estimate of the memory from the cointegrating residuals obtained using a consistent estimator for the cointegrating vector β such as the narrow-band least squares estimator of Robinson (1994), Robinson and Marinucci (2003), and Christensen and Nielsen (2006), among others. In contrast to that, the test of Chen and Hurvich (2006) (denoted by CH06) is directly based on $\hat{d}_v$, but the cointegrating space is estimated by the eigenvectors of the averaged and tapered periodogram matrix local to the origin. Obviously, these two tests are residual-based.

Leschinski et al. (2020) suggest that the three testing procedures have better performance among a group of eight semiparametric tests, particularly when testing for fractional cointegration. In bivariate cases, SRFB18 presents the best performance. Following their finding, we perform all tests at the 5% significance level. The bandwidth is selected as $m = T^{0.75}$ for all three testing procedures. The trimming parameter r is set to 3 for SRFB18 and the integer for averaging the periodogram is 25 for CH06. In our analysis, we test the full sample for stationary fractional cointegration as the order of integration for all series is less than 0.5. As fractional cointegration needs as a core assumption that the order of integration is equal between the series, we use pairwise tests as suggested by Robinson and Yajima (2002) to test for the equality of the memory parameters. The pairs Germany-Italy and France-Italy reject the hypothesis of equality at the 5% significance level and, therefore, are not included in our analysis. The testing period concerns the full sample for two reasons: a) the order of integration is equal and greater than zero, b) the length of the sample can provide more consistent results.

Table 4: Tests for stationary fractional cointegration in inflation

Full sample	SRFB18	WWC15	CH06
Austria	4.454**	22.523**	2.201**
Belgium	2.983**	7.112**	1.467**
Finland	1.829	8.603**	1.140
France	2.110**	9.753**	0.804
Netherlands	2.306**	9.362**	0.92
Greece	1.489	5.950**	1.381
Ireland	2.472**	5.089**	1.709**
Portugal	2.225**	10.495**	1.886**
Spain	2.135**	7.596**	1.210

Note: Critical values at $\alpha = 5\%$ are 1.960 for both SRFB18 and WWC15, as well as 1.386 for CH06. ** denotes significance of the test statistic at the 5% level.

In Table 4 we report the test results for pairwise cointegrating relationships under the null hypothesis of no fractional cointegration. We use Germany as reference country, since it is the largest economy within the EMU. There is evidence for fractional cointegration of inflation rates with the German counterpart for seven out of ten countries in at least two testing procedures. Austria, Belgium, Ireland and Portugal reject the null of no fractional cointegration in inflation for all three tests. Finland and Greece suggest inflation convergence only for the WWC15 test, which is the only one that rejects for all countries. Overall, we find strong evidence of fractional cointegration in inflation over the full sample period, suggesting that even though inflation persistence increased during the crisis period, there still exists a long-run equilibrium among EMU inflation rates.

As a second step, we estimate the reduction rate b of the fractional cointegration relation. The higher b is, the smaller is the difference $d - b$ and thus the less persistent

is the equilibrium process. The standard approach for doing this might be to estimate the cointegration vector β as well as the parameter b by for instance narrow band least squares estimation. We, however, use the property of the Souza et al. (2018) test to allow for estimating b without pre-specifying the cointegrating vector and thus estimating the regression equation. This is due to the purely spectral approach of the test. The advantage of this approach is that a non-rejection of the test cannot alter to an identified cointegration relation due to the separate estimation of the cointegrating vector and the order of integration, which is a common drawback of the narrow band least squares estimation.

Table 5: Cointegration strength pairs in inflation (SRFB18 test)

Full sample	Austria	Belgium	Finland	France	Germany	Netherlands	Greece	Ireland	Italy	Portugal	Spain
Austria		0.39	0.17	0.17	0.38	0.19					0.23
Belgium	0.39			0.26	0.25		0.24		0.14	0.22	0.31
Finland	0.17			0.18		0.25	0.17	0.23			0.25
France	0.17	0.26	0.18		0.18	0.18	0.28			0.26	0.35
Germany	0.38	0.25		0.18		0.20	0.21			0.19	0.18
Netherlands	0.19		0.25	0.18	0.20		0.20	0.30		0.20	0.28
Greece		0.24	0.17	0.28		0.20					0.31
Ireland			0.23		0.21	0.30				0.25	0.30
Italy		0.14								0.23	0.31
Portugal		0.22		0.26	0.19	0.20		0.25	0.23		0.32
Spain	0.23	0.31	0.25	0.35	0.18	0.28	0.31	0.30	0.31	0.32	

We thus repeat the Souza et al. (2018) test between all EMU country pairs and we present the memory reduction parameter in Table 5. Only the cases where SRFB18 and one more or both of the remaining tests are statistically significant are presented. As can be seen in Table 5, the tests suggest that more equilibrium relationships exist for the core countries (upper part of the table), than for the periphery countries.

For the core countries, inflation rates in Austria-Belgium and Austria-Germany are estimated to have the strongest cointegration relation, as the adjustment to equilibrium is achieved faster after potential shocks (b is 0.39 and 0.38, respectively). Additionally, France shares a strong cointegration relationship with Spain (b=0.35). France, Finland and the Netherlands are all estimated to have a large number of fractional cointegration relationships, but with slower adjustment to equilibrium overall.

For the periphery countries, the strongest fractional cointegration relations are found for the inflation pairs Spain-France, Spain-Portugal, Italy-Spain, Spain-Ireland and Spain-Belgium (b is estimated between 0.35-0.30). On average, the cointegration relationships among the periphery countries are somewhat less strong than those among the periphery countries. Interestingly, Greece and Portugal are estimated to have much stronger cointegration relationships with the core countries, than with the periphery countries. A notable exception is Spain, which is found to be cointegrated with all ten countries in our sample.

In summary, we find that inflation persistence since the beginning of EMU is stationary mean-reverting. When we divide the sample into pre- and post-crisis periods, we observe

long memory parameters around zero in the pre-crisis period for the majority of the sample, and substantially higher values around 0.4 in the post-crisis period. Moreover, the inflation persistence during the crisis period increased in particular in the periphery countries, implying potentials for inflation divergence. Our findings are supported by a test in persistence change, showing evidence of breaks in the beginning of the financial crisis. Finally, while we find evidence of fractional cointegration in inflation among the full sample, the core countries tend to be fractionally cointegrated with a faster speed of adjustment, see also (Hualde and Iacone (2017b), Karanasos et al. (2016).

4.2 Analysis of Convergence and Divergence of EMU Industrial Production

4.2.1 Measuring the degree of long memory in EMU industrial production

Following the analysis of convergence in EMU inflation rates in the previous section, we next investigate the real convergence of industrial production between the same countries. First, as in subsection 4.1.1 we estimate the memory parameter of each country. Results are presented in Table 6 for all periods under examination.

Table 6: Memory estimates for industrial production

EMU country	Pre-crisis	Crisis/Post-crisis	Full sample
Austria	0.69	0.74	0.73
Belgium	0.62	0.53	0.61
Finland	0.55	0.40	0.54
France	0.50	0.70	0.70
Germany	0.59	0.80	0.78
Netherlands	0.45	0.53	0.48
Greece	0.55	0.20	0.36
Ireland	0.39	0.28	0.24
Italy	0.54	0.61	0.58
Portugal	0.33	0.59	0.50
Spain	0.66	0.81	0.72
Mean	0.54	0.56	0.56

Note: Exact local Whittle estimates of d with bandwidth $m = T^{0.75}$.

Starting with the full sample, we observe that the majority of the series are nonstationary mean-reverting as $0.5 < d < 1$. However, memory parameters for Greece, Ireland and the Netherlands are below 0.5. While the average degree of long memory of industrial production remained almost constant between the pre- and the post-crisis periods, there are substantial differences among the individual countries. Austria, France, Germany, the Netherlands, Italy, Portugal and Spain all experienced substantial increases in IP long memory, suggesting that real persistence increased considerably during the crisis. By

contrast, the memory estimates are lower during the crisis for Belgium, Finland, Greece and Ireland. This finding implies that some EMU crisis experienced more persistent industrial production during the crisis, while other experienced a higher degree of volatility, i.e. less persistence, during the crisis. Interestingly, this distinction cannot be linked to the core-periphery country groups. Nevertheless, overall there are more cases of memory estimates with $d < 0.5$ in the periphery countries, while in the core countries almost all memory estimates are nonstationary mean-reverting with $0.5 < d < 1$.

Next, we apply the test by Martins and Rodrigues (2014) to test for a break in the order of fractional integration in EMU industrial production. The results are shown in Table 7. As can be seen, most break points occur either around the beginning of the financial crisis in 2007-09 or around the European sovereign debt crisis 2010-13. The test further indicates the end of the crisis period in some countries: For instance, Germany experienced higher persistence in industrial production between December, 2007, and January, 2013. Similarly to the results for inflation, this implies a higher possibility for real divergence of German IP during this period. We find similar results for France and Italy. By contrast, Portugal and Spain are estimated to have experienced higher persistence in IP from 2006/07 until the end of the sample period. As indicated already in Table 6, IP persistence was lower in Belgium, Finland and Greece after the break. The difference is particularly striking in the case of Greece, where IP persistence drops to almost zero after July, 2009. This implies that industrial production in Greece during the crisis years was very volatile. Overall, it thus seems that unlike our results for inflation, we do not always observe more persistent industrial production during the crisis period.

Table 7: Persistence change test for industrial production

EMU country	Test-statistic	Date	d_1	d_2
Belgium	-3.02***	12/2013	0.35	0.27
Finland	-4.21***	06/2010	0.45	0.25
France	-3.17***	01/2008	0.36	0.78
	-2.89***	02/2011	0.69	0.16
Germany	-2.98***	12/2007	0.23	0.42
	-3.75***	01/2013	0.40	0.11
Greece	-1.76***	07/2009	0.33	0.04
Italy	-3.39***	11/2007	0.02	0.56
	-3.81***	09/2010	0.52	0.07
Portugal	-2.03**	06/2006	0.35	0.47
Spain	-2.56*	10/2007	-0.04	0.59

Note: *, **, *** indicate levels of significance at 10%, 5%, 1%, respectively. "d_1" and "d_2" refer to the memory parameters of the respective subperiod of the estimated breakpoint.

4.2.2 Testing for fractional cointegration in EMU industrial production

Finally, we test for fractional cointegration in EMU industrial production among all possible country pairs. We exclude results for Greece, Ireland and Netherlands for two reasons: a) they reject the hypothesis of memory equality with Austria, Germany, France, Italy and Spain and b) they are not fractionally cointegrated with any of the remaining countries. Our results in Table 8 support evidence for cointegration between industrial production in Germany and several EMU countries: There is cointegration with Austria, Finland and France from the group of core countries and with Italy, Portugal and Spain from the periphery countries, respectively. Evidence for strong cointegration holds only for Italy, since $b > 0.5$. It should be noted that for the case of Germany, the WWC15 test does not reject the null of no of fractional cointegration with any other country.

Table again reports estimates of the reduction rate b of the fractional cointegration relation from the Souza et al. (2018) on IP in all country-pairs. Comparing our results for fractional integration of EMU inflation and IP, we find less evidence of real convergence in the sense that less cointegration relationships are found both among the core and among the periphery countries. However, some of the existing long-run equilibria appear stronger than those for inflation, as indicated by the high b, for instance in the country pairs Germany-Italy ($b = 0.68$) and Spain-Italy ($b = 0.64$). Also, while inflation convergence was suggested mostly between core and between periphery countries, convergence in real IP appears stronger between mixed groups of core/periphery countries. For instance, German IP is fractionally cointegrated with Austria and France, but also Italy, Portugal and Spain. This result could mirror the strong trade links between these countries. Overall, it appears that nominal convergence does not necessarily imply real convergence and *vice versa*.

Table 8: Tests for stationary fractional cointegration in industrial production

Full sample	SRFB18	WWC15	CH06
Austria	4.905**	0.245	2.318**
Belgium	2.22**	0.248	1.215
Finland	6.663**	0.044	1.802**
France	5.882**	0.110	3.284**
Italy	9.324**	0.169	2.491**
Portugal	4.256**	0.172	1.994**
Spain	6.381**	0.025	3.146**

Note: Critical values at $\alpha = 5\%$ are 1.960 for both SRFB18 and WWC15, as well as 1.386 for CH06. ** indicates significance at the 5% level.

Table 9: Cointegration strength pairs in industrial production (SRF18 test)

Full sample	Austria	Belgium	Finland	France	Germany	Italy	Portugal	Spain
Austria				0.22	0.36	0.24		0.25
Belgium			0.30					
Finland		0.30				0.46		
France	0.22				0.44	0.47		0.44
Germany	0.36		0.46	0.44		0.68	0.31	0.47
Italy	0.24			0.47	0.68			0.64
Portugal					0.32			
Spain	0.25			0.44	0.47	0.64		

5 Conclusion

In this paper, we apply methods of fractional cointegration to investigate the degree of real and nominal convergence between EMU countries. Specifically, we model both inflation rates and real industrial production indices as fractionally integrated processes and estimate the memory parameters as a measure of persistence. The analysis covers the full EMU period from January, 1999, to June, 2019 for 11 EMU countries consisting of both "core" and "periphery" countries. Moreover, we test for breaks in persistence with the test by Martins and Rodrigues (2014).

Our results suggest breaks in the persistence of both inflation and industrial production around the beginning of the financial crisis in 2007-08 for a majority of countries in our sample. For some countries, we find a second break in 2011-13 (IP) or 2014-15 (inflation) marking the end of the crisis period. Our estimates of inflation persistence show a clear pattern: We find very low inflation persistence before the crisis, substantially higher persistence during the crisis, and again lower persistence after the second break (if it exists). Moreover, inflation persistence is found to be higher for periphery countries than for core countries, in particular during the crisis period. This finding suggests that inflation rates where well integrated in the EMU during the pre-crisis period, while the higher persistence during the crisis period carries the danger of diverging processes in case of asymmetric inflation shocks. By contrast, the results on time-variation in persistence of real IP are less clear. Overall, there is evidence of higher persistence in the core, than in the periphery countries. This is in contrast to the results for inflation persistence. Regarding the time-variation in IP persistence, we find that the majority of countries experienced higher real persistence during the crisis period. However, Belgium, Finland and Greece show the opposite pattern with lower persistence during the crisis.

Next, we test for the existence of fractional cointegration relationships for inflation and IP among all country pairs of our sample and estimate the reduction rate b of the fractional cointegration relation using the Souza et al. (2018) test for the full sample. We find evidence of fractional cointegration among EMU countries for both inflation and real IP. In the case of inflation, the estimates suggest mostly convergence with weak persistence, but we find more cointegration relations overall than for industrial production. Convergence "clubs" as indicated by cointegration relations emerge primarily among core and among periphery countries. In the case of industrial production, we find fewer cointegration relations among the country pairs, but these are overall stronger than those for inflation. Moreover, we now find mixed core-periphery convergence clubs, potentially due to trade links among these countries.

To sum up, our analysis gives a detailed picture of time-variation in real and nominal convergence processes since the start of the EMU. While the financial crisis and the European sovereign debt crisis was a period of potential divergence in both inflation and industrial production, there is nevertheless some evidence that a) the crisis period ended in

many cases before the end of our sample and b) persistence increased during the crisis, but we still find evidence of fractional cointegration for the full sample period. These positive news notwithstanding, the result that the strength of fractional cointegration relationships for inflation is much lower than for industrial production tells a cautious tale about the potential vulnerability of nominal convergence and the dangers of inflation differentials within the monetary union. Since inflation differentials can lead to differences in real rates and, therefore, in competitiveness among the EMU members, this issue remains an important factor for policy makers in EMU also after the European sovereign debt crisis.

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